PRIMARY ADVANTAGE

Maths Programme

A Model of Best Practice

Third Edition

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PRIMARY ADVANTAGE
SCHOOLS ACHIEVING MORE TOGETHER

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Further information

For further information about the issues discussed in this publication please contact:
teachingschools@primaryadvantage.hackney.sch.uk

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Introduction

**At the individual level, mathematics underpins r
activities, from making sense of information in t
about personal finances.**

substantial amount of time is devoted to the teaching and learning of mathematics to build a strong foundation for the acquisition of mathematics knowledge and skills in later years.

Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject. Achieving mastery means acquiring a solid enough understanding of the maths that's been taught to enable pupils to move on to more advanced material. (NCETM 2016)

At Primary Advantage, we want our pupils to truly master mathematical skills, to have the strongest understanding and experience applying the skills they have learnt across a wide range of contexts.

We want to ensure the learning has "stuck". Through systematically teaching the objectives within the programme, alongside our assessment model of Prove Its (low stakes assessment activities designed to ensure that previously mastered learning is regularly revisited) we ensure that pupils have the opportunities they need to connect the mathematical ideas together.

Effective learning of mathematics requires a coherent and well-structured syllabus, excellent instructional materials, and excellent teachers who use sound pedagogical strategies that are developmentally appropriate. This is at the core of the Primary Advantage Programme. Our mathematics curriculum emphasises conceptual understanding, skills proficiency, learning of process skills and focuses on mathematical problem solving.

In sharing our experience in the development of this maths curriculum we hope to have made a positive contribution to Primary Mathematics teaching.

Acknowledgements: The programme was developed by a group of teachers from Primary Advantage Schools and without their expertise and professionalism the syllabus could not have been completed. We would like to thank Gemma Meharg, Catherine Thomas, Stephanie Saviddes, Sarah Jameson, Joanne Smith, Jo Stonehouse, Aidan Stallwood and Alyson Tyler for their input and positivity throughout the project.

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Aims and distinctive features

This programme aims to support you in developing the
knowledge.

Mathematical knowledge

The programme and CPD will support in developing understanding of mathematics

- **Curriculum knowledge**

By following the programme you will come to learn exactly which areas of mathematics should be taught to each group of children you may be working with

- **Pedagogical knowledge**

The programme draws on models and images so that you can see the best ways to introduce learners to particular mathematical ideas.

The programme is practical and models best practice in primary mathematics teaching.

It will support you in the planning and delivery of lessons for the full primary range.

Progression

The learning progressions within each strand are built on each other. This will help you understand which mathematical ideas should be taught to which age group of children and how the Mathematics Curriculum is developed over the primary age range.

Key concepts

These sections deal with the ideas which underpin each particular strand of mathematics covered. This allows you to see the big picture immediately and understand how the different strands knit together.

Models and images

These sections deal with the best models and images to represent the elements of mathematics in each strand. This will help you in choosing appropriate representations when planning mathematics lessons. Examples of the bar model will help you when planning for problem solving in your class.

The overview of the PA Maths Programme

Programme aim

The PA maths Programme aims to empower young people to achieve their potential, to use their knowledge of mathematical language to talk about their work and explain their findings, and ultimately use the skills they have mastered to make informed and responsible choices throughout their lives.

Infusing

Curriculum Objectives Fluency in the fundamentals of mathematics Reason mathematically Solve problems

Cross Curricular Skills COMMUNICATION USING MATHEMATICS ICT

Thinking Skills and Personal Managing information, working with others Thinking, problem solving, Decision Making, Self-management Being creative **Capabilities**

Incorporating

Assessment for Learning Cleared learning intentions shared with pupil

Promoting/Encouraging
Shared/negotiated success criteria

Ownership of learning Taking risks for learning how to achieve
Peer and self assessment / evaluation of learning
Celebrating success Advice what to improve and

Supportive environment

Relevant and enjoyable

Commitment

Active and hands on

Culturally diverse

On—going reflection

Challenging and engaging

Respect

Learning Experiences

Fostering

Media—rich

Investigating and problem solving

Concern for others

Positive reinforcement

Skills integrated Varied

Attitudes and Dispositions

Open to new ideas

Personal responsibility

Offers choice Enquiry based

Curriculum links

Determination

Self-confidence

Curiosity Collaboration

Flexibility

Resourcefulness

Resilience

Primary Advantage Learning Dispositions

Learning mathematics extends beyond learning concepts, procedures, and their applications.

It also includes developing a disposition toward mathematics and seeing mathematics as a powerful way for looking at situations.

Disposition refers not simply to attitudes but to a tendency to think and to act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks – whether with confidence, willingness to explore alternatives, perseverance, and interest – and in their tendency to reflect on their own thinking. The assessment of mathematical knowledge includes evaluations of these indicators and students' appreciation of the role and value of mathematics.

As part of the Primary Advantage programme we encourage and reward the following standards in all of our lessons.

Primary Advantage CPD

This innovative programme is designed to support teachers in developing children's mathematical understanding and enjoyment throughout the primary phase. It has been designed by a range of classroom teachers and is a model of best practice, based on experience and theoretical understanding.

The PA Maths Programme comprises of a range of complementary elements which all contribute towards the aim of high quality primary maths teaching and learning:

- ξ 4 core training CPD sessions
- ξ Induction training for staff new to using the programme
- ξ 'How to' sessions, TA subject knowledge sessions and EYFS training sessions
- ξ The opportunity to attend modelled lessons within the PA schools
- ξ Access to the online Maths Portal (including this supporting document)

Through engaging with the programme, teachers' planning and lesson delivery will develop in the following ways:

Mathematical knowledge	A deeper understanding of the underpinning structures of primary mathematics.
Curriculum knowledge	A secure understanding of best practice in terms of progre and challenge through curriculum planning.

Pedagogical knowledge	A deeper understanding of important models and images to support children's progress through a range of key mathematical concepts.

By engaging with the PA Maths Programme, teachers are demonstrating a commitment to sharing the values that underpin it, namely the belief that primary mathematics is a crucial phase in laying the foundations for lifelong numeracy skills. This drives the focus on conceptual understanding, skills proficiency, problem solving, reasoning and fluency which comprise the PA Maths Programme.

Through sharing our experiences with the wider primary community through the PA Maths Programme, we hope to make a positive contribution towards maths teaching and learning and thank you for joining us on our journey.

Problem Solving, Reasoning and Fluency

As its underpinning aims, problem solving, reasoning and fluency are at the heart of the National Curriculum for England and Wales (DfE, 2014). By highlighting them in this way, the DfE have indicated that they should underpin the curriculum by threading through all of the teaching and learning. By using them as a lens through which to teach the content, the national curriculum will be taught in its intended manner.

The PA Maths Programme supports this approach and believes that through developing children's problem solving, reasoning and fluency skills, there will be a range of positive outcomes, including the development of children's conceptual understanding, their ability to use maths in meaningful ways and positive attitudes from the EYFS to Year6.

Problem Solving

This can be summarised as the ability to apply mathematics to a variety of situations (Cockcroft, 1982) and PA encourage the use of 'low threshold, high ceiling' activities. These mathematical activities are designed so that the great majority of the group can begin, and then work on at their own level of engagement, but which has lots of possibilities for the participants to do much more challenging mathematics (McClure, ND). They can lead to the development of a community of practice, positive attitudes and progression through deepening subject knowledge, rather than accelerating. There are a wealth of activities on the NRich website (www.nrich~~.maths.org.~~), ~~alongside~~ others, which can be used alongside this PA document.

Reasoning

Reasoning can be considered the glue which holds maths together. A focus on the mathematical process and a real commitment to children's understanding, as distinct from any final product, enables the development of reasoning skills in the primary classroom. The 'example questions' section of the pages in this document help support this aspect of teachers' planning and, when used to encourage children to move from describing to explaining to justifying, it is another tool with which to challenge the higher attaining children.

Fluency

Developing children's mathematical fluency demands a focus on their efficiency, accuracy and flexibility. It requires them to know why they are doing what they are doing, and to make appropriate choices (from a toolkit of mental calculation strategies, for example). By using manipulatives (as part of a CPA approach), encouraging children to discuss their work, particularly through reasoning, and consolidating understanding across a range of meaningful contexts, children's fluency skills will develop.

These three aims are inter-related and complementary. They inform the PA Maths Programme and are deemed to be at the root of high quality maths

teaching and learning.

Cockcroft, W H (1982) *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*, London, Her Majesty's Stationery Office

Why?

CPA in the PA Maths Programme

Another key feature of the CPA process, is that although concrete objects

A commitment to CPA is intrinsic to the PA Maths Programme. It informs the pedagogy and planning of teachers using this programme as PA believe it is a supportive way of developing children's deep conceptual understanding, good progression and positive attitudes to maths.

What?

CPA is an approach to teaching mathematics based on the work of Jerome Bruner (1960). Bruner's premise was that children's conceptual understanding develops from being actively engaged in their learning and making sequential process through three stages of representation: enactive, iconic and symbolic (mapped onto **c**oncrete, **p**ictorial, **a**bstract respectively). Each stage builds on the previous one, although unlike Piagetian theory, they are not age-related.

CPA therefore encompasses multiple models that approach a concept at different cognitive levels. Firstly at the concrete level, children are exposed to a range of appropriate manipulatives, for example, dienes, unifix, Numicon, egg boxes, counters, shapes, coins and dice. Use of these concrete objects engages children with their learning and can provide a 'hook' into the learning. Another advantage of this approach is that discussion is a natural by-product of active learning which is an element of good quality maths teaching and learning (Williams, 2008).

Progress into the pictorial phase is consequently underpinned by active, memorable experiences leading to deep learning. This second phase aids visualisation and the bar model is a key element of the pictorial phase of problem solving (this is explored later).

It is important to note that although the ultimate aim of a CPA strategy is to culminate in a fluent, abstract approach characterised by quick, efficient methods, the process should not be rushed. It may be necessary to return to previous phases to address children's misconceptions and consolidate their conceptual understanding.

may be perceived as too elementary for upper KS2 children (Sousa, 2007), both concrete and pictorial representations should be used at across the primary phase.

How?

Within the PA core CPD sessions, explanations and examples of appropriate CPA apparatus, models and images are consistently discussed. They also form the backbone of this document as each year group has clear diagrams demonstrating the progression from the concrete to pictorial to abstract. This structure informs teachers' planning and pedagogy, as does reflection on observations of any modelled lessons attended.

Two key facets of the CPA approach (the counting stick and the bar model) are discussed in depth in the following pages.

CPA Exemplified

Whilst choices around which concrete and pictorial representations are used within lessons must remain the choice of each teacher, dependent upon their individual context, there are two which PA would advocate as useful across a range of

learning experiences – the counting stick and the bar model.

Counting stick

This piece of concrete apparatus embodies the ubiquitous pictorial model of the number line. Traditionally it has been used for counting on/back in ones/tens and for ordering numbers, but the counting stick is a versatile piece of equipment which can be used across a range of mathematical areas to support children's fluency and understanding.

A focus on fluency is often the genesis of counting stick use, for example children counting forwards and backwards in multiples of a given number. However, by labelling the ends of the counting stick with two numbers and asking children to label another given point, their reasoning skills are required. Careful questioning can encourage them to move from giving a basic answer to explaining and then justifying their thinking. Contextualising this by using a counting stick vertically to represent scales of temperature, length or mass can be engaging and meaningful for children.

Ideas of equivalence can be developed and consolidated through using a number line too as children begin to explore other possible labels for given points such as ¼, 0.25 and 25%. The counting stick frequently features in the models and images pages throughout this document.

Bar Model

The 'bar model' is shorthand for a systematic method of representing word problems and number relationships. It is exemplified by children sketching rectangular bars to represent relationships between known and unknown numerical quantities. In this way, it can help children identify which calculation is needed to solve a word problem or investigation. It is often this step which is problematic for children, rather than the calculation itself, so the bar model, and the discussion which supports it, can be very useful.

E.g. Ali has £10 to spend on books. He chooses one for £6 and one for £1.99. How much change does he get from his £10 note?

£6

The bar model is a visual, flexible strategy which children can fit into their 'toolbox' of heuristics for problem solving. It can be supported by concrete apparatus such as Cuisenaire rods or unfix cubes, to enable children to access and enjoy mathematical problem solving. The bar model frequently features in the models and images pages throughout this document.

Yearly Overviews

		EYFS 1						EYFS 2					
Order		1	2	3	4	5	6	7	8	9	10	11	12

		To make comparisons between quantities. ***NPV, A & S*** To use some language of quantities, such as 'more' and 'a lot'. ***NPV & A***	To use the language of more and fewer (less) to compare sets of objects. ***A & S*** To recite number names in sequence to 10. (0-10) ***NPV***	Select a small number of objects from a group 'give me one', 'give me two'. ***NPV*** To compare two groups of objects (identifying 'the same'). ***NPV*** To use number names and language. (to recognise numbers) ***NPV***	To separate a group of three or four objects in different ways. (Total is still the same) ***A & S*** To know that numbers identify how many objects are in a set. (Triad) ***NPV*** To know that a group of things changes in quantity when something is added or taken away. ***S***	To match numeral and quantity.***NPV*** To use one to one correspondence (touch each object and give it a number 0-10) ***NPV*** To count objects in a line. ***NPV*** To create and experiment with symbols and marks representing number. ***NPV*** To respond to (and use) addition vocabulary in rhymes and games. ***A*** To respond to (and use) subtraction vocabulary in rhymes and games. ***S***	To use more/most and less/least. ***NPV*** To find the total number of items in two groups, after some are added, by counting all of them. ***A*** To find the total number of items, after some are taken away, by counting all of them. ***S*** To know that when counting a group the last number represents the quantity. ***A & S***	To count reliably (from 0-20) ***NPV*** To count objects to 10, and beginning to count beyond 10 *(Can count in a line)* ***NPV*** To use one to one correspondence (touch each object and give it a number 0-20) ***NPV*** To count actions or objects which cannot be moved. ***NPV*** To count objects in a group/ irregular arrangement of up to ten objects (same group/different group). ***NPV*** To represent numbers using fingers, marks on paper or pictures. ***NPV*** To recognise numerals. (0 to 5, 0-10 & 0-20)***NPV*** To order numbers to 20. ***NPV***	To write numbers to 20. ***NPV*** To find/ say the number which is one more or one less than a given number. ***A & S*** Relates addition to combining two groups. ***A*** Relates subtraction to taking away. ***S*** To find **one more** or **one less** from a group of up to five objects, then ten objects. ***A & S*** Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. To set out groups and find the total amount. ***Mx***	To estimate how many objects they can see and check by counting. ***NPV*** To recognise the number of objects in a small group without counting out (subitise). ***NPV*** Uses quantities and objects, to add two single-digit numbers and **count on** to find the answer. ***A*** To count on when adding to a group (holding first number in head) ***A*** To add two sets of objects which are the same (cars + cars) then different (apples + bananas) ***A*** Increase one quantity by a given amount to find the total (augmentation) ***A*** To share objects equally. ***D*** Records, using marks that they can interpret and explain. ***A & S***	To use quantities and objects, to subtract two single-digit numbers (count on or back) to find the answer. ***S*** To count backwards.(on a number line or counting stick.) ***S*** To recognise and name +, =, - signs. ***A & S*** To read an addition number sentence. ***A*** To solve an addition number sentence. ***A*** To read a subtraction sentence. ***S*** To solve a subtraction number sentence. ***S*** To group objects. ***D***	To make 10 (feel the ten ness of ten). ***NPV*** To arrange an addition number sentence. ***A&S*** To arrange a subtraction number sentence. ***S*** To halve (an even group up to 12) ***S & D*** To solve problems involving grouping and sharing. ***F*** To share an even group of objects between 2, between 4. ***D & F*** To skip count in 2s, 5s & 10s. ***Mx*** Begin to understand odd and even. ***Mx & D*** To count up to 20 (objects/ images in an array) ***D***	Shares an even group of objects between 4. ***D*** To know number families to 5, 6 & 10. ***A & S*** To begin to identify own mathematical problems based on own interests and fascinations. ***S*** To know doubles to 10. ***A*** Begin to relate the addition of dobles to counting on (how many wheels on 2 cars? 4… 5,6,7,8 4+4=8) ***Mx***
Key: Number and Place Value ***NPV***, Addition ***A***, Subtraction ***S***, Multiplication ***Mx***, Division ***D***, Fractions ***F*** and Measurement ***M***														

EYFS 1						EYFS 2					
1	2	3	4	5	6	7	8	9	10	11	12
	Can say what is different and what is the same. ***M***	Begins to categorise objects according to properties such as size (colour.) ***M*** Begins to categorise objects according to properties such as shape. ***GS***	Begins to use the language of size. ***M*** Shows an interest in shape and space by making arrangements with objects. ***M***	Experiments with capacity. (Which holds more/less) ***M*** Begins to talk about the shapes of everyday objects, e.g. 'round' and 'tall'. ***GS***	Anticipates specific time based events such as mealtimes or home time. ***M*** Understands some talk about immediate future, e.g. 'later' or 'soon'. ***M*** Understands some talk about immediate past e.g. 'before'. ***M*** Uses money in role play. ***M***	Exchanges money for objects. ***M*** Shows awarenes s of similaritie s of shapes in the environm ent. Uses familiar objects and common shapes to build models. ***GS*** Beginning to use mathematical names and 'flat' 2D shapes. ***GS***	Uses positional language (below, above, next to, beside, in front, behind and on top) ***GP***	Describes their relative position such as 'behind' or 'next to'. Uses mathematical terms to describe 2d shapes. ***GS***	Orders two items by mass. (using everyday language) ***M*** **Uses everyday language to solve problems. *M*** **They recognise, create and describe patterns.** To count patterns. ***Mx*** Orders two or three items by length or height. ***M***	Orders two items by capacity. (using everyday language) ***M*** **Uses everyday language to compare quantities & objects. *M*** **Uses everyday language to talk about distance. *M*** Orders and sequences familiar events. ***M*** **Uses everyday language related to time (begins to identify o'clock) *M***	Measures short periods of time in simple ways. ***M*** **Uses everyday language to talk about money. *M*** Demonstrates understanding that £1 has greater value than pennies. ***M***	Know and name different coins – 1p, 2p, 5p. 10p, 20p, 50p, £1 & $2. ***M*** Can use 1p, 2p, 5p & 10p coins to make amounts up to 20p. ***M*** To identify half a shape. ***F*** To put together halves to make whole shapes. ***F*** To break an object in half. ***F*** Uses mathematical terms to describe 3d shapes. ***GS***
	Key: Number and Place Value ***NPV***, Addition ***A***, Subtraction ***S***, Multiplication ***Mx***, Division ***D***, Fractions ***F*** ,Measurement ***M,*** Geometry Shape ***GS,*** Geometry Position ***GP***										

MATHEMATICS YEARLY OVERVIEW
YEAR 1

Term	1	2	3	4	5	6	7

Autumn	**Number and place value**	**Number and place value**	**Addition**	**Addition**	**Subtraction**	**Subtraction**	
	Number and place value	**Measure –** Money	**Addition and Subtraction** *(context money)*	**Measure –** Length	**Addition and Subtraction** *(context length)*	**Geometry –** Properties of shapes	**Statistics**
Spring	**Measure -** Time	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Measure –** Capacity and Mass	**Addition and Subtraction** *(context capacity and mass)*	
	Geometry – Position and direction	**Addition**	**Multiplication**	**Division**	**Fractions**		
Summer	**Measure –** Time	**Geometry –** Properties of shapes	**Number and place value**	**Measure –** Money	**Addition/ Subtraction**	**Addition/ Subtraction**	
	Addition/ Subtraction	**Addition/ Subtraction**	**Multiplication**	**Multiplication**	**Division**	**Fractions**	**Statistics**

MATHEMATICS YEARLY OVERVIEW
YEAR 2

Term	1	2	3	4	5	6	7

Autumn	**Number and place value**	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Measure -** Time	
	Geometry – Properties of shapes	**Fractions**	**Division**	**Multiplication**	**Statistics**	**Measure –** Money	**Addition and Subtraction** *(context money)*
Spring	**Number and place value**	**Addition/ Subtraction**	**Addition/ Subtraction**	**Multiplicatio n/ Division**	**Multiplicatio n/ Division**	**Measure -** Time	
	Measure – Length	**Addition and Subtraction** *(context length)*	**Multiplicati on and Division** *(context length)*	**Fractions**	**Geometry –** Position and direction		
Summer	**Geometry –** Properties of Shapes	**Number and place value**	**Measure –** Capacity and Mass	**Addition and Subtraction** *(context capacity and mass)*	**Multiplicati on and Division** *(context capacity and mass)*	**Measure –** Time	
	Measure – Length	**Four Operations** *(context measure)*	**Four Operations** *(context money)*	**Fractions**	**Statistics**	**Geometry –** Position and direction	**Four Operations** *(context measure)*

MATHEMATICS YEARLY OVERVIEW YEAR 3

Term	1	2	3	4	5	6	7

Autumn	**Number and place value**	**Addition and Subtraction**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Measure –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry** – Properties of shapes	**Statistics**	**Measure –** Volume and capacity	**Measure –** Length and mass	**Four Operations** *(context volume, capacity, length and mass)*
Spring	**Number and place value**	**Geometry** – Properties of shapes	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Addition and Subtraction**	**Multiplication and Division**	
	Statistics	**Measure –** Money	**Four Operations** *(context money)*	**Measure –** Time	**Four Operations**		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Measure – Volume and capacity	**Four Operations** *(context volume and capacity)*	**Measure –** Length and mass	**Four Operations** *(context length and mass)*	**Geometry** – Properties of shapes	**Statistics**	**Measure –** Time

MATHEMATICS YEARLY OVERVIEW
YEAR 4

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry** – Properties of shapes	**Statistics**	**Measurement –** Length and mass	**Measurement –** Volume and capacity	**Four Operations** *(context volume, capacity, length and mass)*
Spring	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Geometry** – Position and direction	
	Statistics	**Measurement –** Money	**Four Operations** *(context money)*	**Measurement –** Time	**Geometry** – Properties of shapes		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Geometry** – Position and direction	
	Measurement – Volume and capacity	**Four Operations** *(context volume and capacity)*	**Measure –** Length and mass	**Four Operations** *(context length and mass)*	**Geometry** – Properties of shapes	**Statistics**	**Measurement –** Time

MATHEMATICS YEARLY OVERVIEW
YEAR 5

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Four Operations**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes	**Measurement –** Length and mass	**Measurement –** Volume and capacity	**Four Operations** *(context volume, capacity, length and mass)*	**Statistics**
Spring	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Measurement – Money	**Four Operations** *(context money)*	**Measurement –** Time	**Geometry –** Position and direction	**Geometry –** Properties of shapes		
Summer	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	
	Statistics	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Measurement –** Volume and capacity	**Measurement –** Length and mass	**Measurement –** Money	**Four Operations** *(measurement)*

MATHEMATICS YEARLY OVERVIEW
YEAR 6

Term	1	2	3	4	5	6	7
Autumn	**Number and place value**	**Addition and Subtraction**	**Multiplication and Division**	**Multiplication and Division**	**Four Operations**	**Measurement –** Time	
	Fractions, Decimals and Percentages	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes	**Measurement –** Volume, capacity and mass	**Measurement –** Length and money	**Four Operations** *(measurement)*	**Statistics**
Spring	**Fractions, Decimals and Percentages**	**Fractions, Decimals and Percentages**	**Four Operations**	**Four Operations**	**Algebra**	**Assessment Week –** move accordingly	
	Four Operations *(context money)*	**Measurement –** Time	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Ratio and Proportion**		
Summer	**Number and place value**	**Four Operations** *(money)*	**Algebra**	**Fractions, Decimals and Percentages**	**Geometry –** Properties of shapes **Ratio and Proportion**	**Statistics**	
	Four Operations	**Geometry –** Position and direction	**Geometry –** Properties of shapes	**Measurement –** Volume, capacity and mass	**Measurement –** Length and money	**Four Operations** *(measurement)*	**Assessment Week –** move accordingly

National Curriculum Strands

Chapter 1

Number and Place Value

EYFS 1 – Number and place value (When planning ensure you track forwards to year 1)	
Early Learning Goal 11 Children count reliably with numbers from 1 to 20 and place them in order.	
Key Vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between. **Key concepts** When there are more objects the group gets bigger. When there are fewer (less) objects the group gets smaller. When counting a group the last number represents the quantity. Anything can be counted: claps, steps, jumps… There are many numbers in the world around us. When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.	**Learning objectives** (see overleaf for exemplification) To make comparisons between quantities. To use language of quantities such as 'more' and 'a lot'. To use the language of 'more' to compare sets of objects. Recite number names in sequence 0-10. Select a small number of objects from a group 'give me one, two etc'. To compare two groups of objects (identifying 'the same'). To use number names and language. To match numeral and quantity correctly. To use one to one correspondence (touches each object and gives it a number). To count objects in a line.

Potential barriers/misconceptions Pupils show confusion in vocabulary- more / less. Misconception can occur through the linking of words- the bigger a number (in size) the greater it's quantity. i.e. 3 is bigger than 7. (worth more than…) Some pupils at this stage cannot differentiate between numbers and letters. Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence. There may be little understanding of the value that each number holds Pupils find it challenging to identify 'same' and 'different when working visually as they don't focus on the detail.	To know that numbers identify how many there are in a set (triad). To create and experiment with symbols and marks representing ideas of numbers, then numerals. To use more/most and less/least.
Example Questions If we count around the circle starting with Gemma, who will say 5? Look at the bowl of apples. Are there more green apples or red apples? How can you find out? The birthday card has a 4 on it. Raza is four today. Put the right number of candles on his birthday cake. How many counters are there? Select the correct number card and match it withthe counters. 1 2 3 4 5 Count with me to 10. One, two, three…. Count these buttons. You can move them as you count them if you wish. What number is the one before six? Put three coats up on the pegs Bring me five aprons. Can you put one back? Are there more books on the top shelf or on the bottom shelf? How do you know? Which set has more cubes? The set of red cubes or the set of green cubes? How do you know? Show a card. 'Read me the number on this card'. Look at this telephone. Can you press the number 4? Number 6?	**Mental maths**(can revisited throughout day once concept has explicitly shared) One, two, three four five. Once I caught a fish alive… One potato, two potatoes, three potatoes, four… Higgledy, piggledy, my fat hen… This old man, he played one… Recite sequence 1,2,3 up to 10 Count objects: tiny things in a matchbox, pieces of a jigsaw, letters in your name etc. Count in 2s: pairs of socks, pairs of animals Count in 1s (say aloud every other number) Recognise recitation errors: (could use a puppet) One, two, four, five (word omitted) One, two, four, three, five (words in the wrongorder) One, two, three, three, four (repeating a word) Start from a given number name and stop at another (start with three, hold it in your head, count to six) Recite the number names in order to 5 then 10 To count backwards from 10 To count backwards to zero (none) from any number Estimate (guess) how many marbles in the jar, coins in a purse etc

EYFS 1 – Number and place value Progression (a combination of these models and images can be used for every objective)

To make comparisons between quantities.	To use language of quantities such as 'more' and 'a lot'.	To use the language of more to compare sets of objects.
Which plate would you like? Why?	I need more cars I have a lot of cars	Which bowl has more fish in?

Recite number names in sequence 0-10.	Select a small number of objects from a group 'give me one, two etc'.	To compare two groups of objects (identifying 'the same').	To use number names and language.
			zero – none six one seven two eight three nine four ten five more, less bigger, smaller
To match numeral and quantity correctly	**To use one to one correspondence (touch each object and give it a number)**	**To count objects in a line**	
To know that numbers identify how many there are in a set. (triad- three key elements)	**To create and experiment with symbols and marks representing ideas of numbers then numeral.**	**To find more/ most and less/least**	
It looks like: It sounds like: **SIX** You make it like this:	Make own marks or tallies resulting from practical activities. Pictorial representations of groups. When beginning to record numbers: Trace with a finger cut out numerals Sandpaper Rough fabric Tin foil Corrugated card Make numbers in sand, foam, using plasticine.	**Isaac's bears Hussein's bear** Who has the most bears? Who has the least?	

EYFS 2 – Number and place value (When planning ensure you track forwards to year 1)	
Early Learning Goal 11 Children count reliably with numbers from 1 to 20 and place them in order.	KS1 ready: Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number. Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s Given a number, identify one more and one less Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least Read and write numbers from 1-20 in numerals and words

Key vocab: number, zero, one, two, three etc, none, how many?, count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, guess how many, estimate, nearly, close to, about the same as, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between.

Key Concepts.

When there are more objects the group gets bigger.
When there are fewer objects the group gets smaller.
When counting a group the last number represents the quantity.
Anything can be counted: claps, steps, jumps…
There are many numbers in the world around me.
We can write number with words and squiggles.
When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.

Learning objectives (see overleaf for exemplification)

To count reliably from 1-20
To use one to one correspondence (touch each object and give it a number)
To count objects in a line- beginning to count beyond 10
To count actions or objects without physically touching them.
To count objects in a group/ irregular arrangement. (using first same objects/ then different objects) To represent numbers using fingers, marks and pictures.
To recognise numerals (0-5), (0-10) and (0-20).
To order numbers from 0-20.
To select the correct numeral to represent 1-5 then 1-10 objects.
Write numbers to 20.
To estimate how many objects can be seen and check by counting.
To recognise numbers in a group without counting out (subitise).
To make ten (feel the ten-ness of ten).

Potential barriers/misconceptions

Pupils show confusion in vocabulary- more / less.
Misconception can occur through the linking of words- the heavier object is the one that is 'higher' on the balance (When using balances to compare quantity).
Pupils may be able to recite number words up to ten but do not count objects with 1 to 1 correspondence. There may be little understanding of the value that each number holds

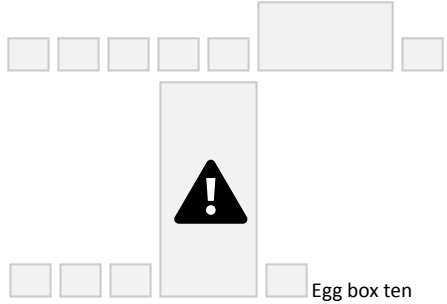
Example Questions

One, two, buckle my shoe, three, four, knock at the door. Which two numbers come next? 10, 9, 8, 7, carry on counting backwards until 'blast off!'
If we count round the circle starting at Lewis with 3, who will say 9?
What number comes next after 12 when you count?
Make a line of toy cars. Make the second car yellow and the fifth car red.
Count on for me as far as you can go. One, two, three… What is the next number after four? I will say some numbers. I want you to count on the next three numbers. Four, five, six…
Take this box of unifix. Count out nine of the unifix cubes and put them onto the table. Can you check? How many circles are there in the picture (show up to 5)? Tell me without counting them. Now check by counting. Guess how may cars there are on the table. (Place up to 10). Now check by counting them. Which plate has fewer biscuits on it? How do you know?
There are 8 cubes in this stick of cubes. There are five cubes in this stick of cubes. Which stick has more cubes? Ella has five apples, Dom has three apples. Who has fewer apples? Ella or Dom?
Choose two cards from this set. Which of your two numbers is more? Which number is less?

Mental maths (can revisited throughout day once concept has explicitly shared)

To count from 1-20
To count from non-zero starting point (up to 20).
To recite the number names in order, continuing the count from a given number
Recognise recitation errors:
(could use puppet) Thirteen, fourteen, fiveteen (not changing the pattern)
Eighteen, nineteen, tenteen (error by analogy)
Thirty-nine, thirty-ten (error by analogy)
Start from a given number name and stop at another. (start with 2, hold it in your head, count on to 8) Count on several numbers from a given number (using fingers to help: count on three numbers from 4) To use ordinal numbers in different contexts (Who is third in the line?)
To say the number name that goes before a given number name. (What number comes before □?) Recite the number sequence consistently back to zero from any given number to 20
To count in 2s, 5s and 10s.
To count on in tens from any given tens number. (Count on in tens from 30)
To count back in tens from any given tens number.
Estimate the number in a group (how many children in class today?)
To know which number is worth more/less.
To say the number that is one more/one less than the given number.
Say a number lying between two given numbers.
Begin to use and understand ordinal numbers in different contexts (first, second, third….. & last)

EYFS 2 – Number and place value Progression (a combination of these models and images can be used for every objective)			
To count reliably from 1-20	**To use one to one correspondence (touch each object and give it a number)**	**To count objects in a line (beyond 10)**	**To count actions or objects without physically touching them.**

one eleven two twelve three thir**teen** four four**teen** five fif**teen** six six**teen** seven seven**teen** eight eigh**teen** nine nine**teen** ten twen**ty**		 **11** There are eleven cars in the group	Listening to the number of claps. The rings of a bell. Jumps in the air. Children at the front of the class. Windows on a building
To count objects in a group/ irregular arrangement	**To represent numbers using fingers, marks and pictures**	**To recognise numerals (0-5), (0-10) and (0-20)**	**To order numbers from 0-20**
 There are nine cars in the group		 'Put the silver car in space 8'	
To select the correct numeral to represent 1-5 then 1-10 objects	**Write numbers to 20**	**To estimate how many objects can be seen and check by counting**	**To recognise numbers in a group without counting out (subitise)**	**To make ten* (recognise the ten-ness of 10)** (*see addition for number bonds within 10)
	Trace. Write in the air. Paint. Model in play dough. Make number signs for classroom.	 How many oranges do you think there are? Can you check?		 Egg box ten Unifix ten bundles of ten straws

Year 1 – Number and place value (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number. Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s
Given a number, identify one more and one less
Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
Read and write numbers from 1-20 in numerals and words

Notes and guidance (non statutory)

Pupils practice counting (1,2,3…) ordering (e.g. first, second, third) and to indicate a quantity (e.g. 3 apples, 2 centimetres), including solving simple concrete problems until they are fluent.
Pupils begin to recognize place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations.
They practice counting as reciting numbers and counting as enumerating objects, and counting in 2s, 5s, and 10s from different multiples to develop their recognition of patterns in the number system (e.g. odd/even) including varied practice through increasingly complex questions.
They recognize and create repeating patterns with objects and with shapes.

Key vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way. **Key Concepts**

A group of objects can be partitioned in a number of ways- the total stays the same. (conservation) Ordinal numbers are for describing the position in a group of objects.
When comparing we use the terms 'greater than' and 'smaller than' and 'more than' and 'fewer than.' When we are talking about objects we say 'more than' and 'fewer than'. When we are talking about numbers we say 'greater than' and 'smaller than'.
Making ten first supports with number conservation. Children then count on from ten rather than starting at 1.

Potential barriers/misconceptions

Unable to recognise numbers.
Knowledge of saying numbers out loud with no concern for value or amount of objects. No understanding of the value that each digit holds.
Understanding of number size – confusion over 3 is bigger than 1.
Does not count with 1-1 correspondence.
Able to count forwards but struggles to count backwards or find 'one less than…'
Counts all rather than counting 'on' (no conservation of number).
Sees a 'ten' as one rather than ten ones.
Confusion between 'teen' numbers and multiples of ten: 16, 60.
Reversal of digits.

Example Questions

What number comes after 22? Before 65?
Count back from 10 to six. How many did you count?
Which tens number comes after 50? Before 80?
What would be the best way to count marbles into the jar?
There is always 1 left over when an odd number is divided by 2. True or false? Can you prove it? Draw a ring around the person who is 9th in the line.
Estimate the number of pencils.
Estimate how many pairs of socks you could make. (Show a picture of unpaired socks)
Look at these numbers: 34 12 45 60 72 28 Which of these numbers is the largest? Which of these numbers is between 10 and 20?
This sentence is correct: 8 is less than 10. Two of these sentences are correct. Tick them: 18 is more than 30, 26 is less than 60, 50 is more than 17, 47 is less than 21.
Fill in the blanks: 35 is more than □, 35 is between □and □, 35 has □ tens.
Write the number thirty-two.
Fill in the missing numbers: 18 is 1 less than □ , 18 is 10 less than □.

Learning objectives (see overleaf for exemplification)

To identify one more and one less.
To compare quantities (using equal to, more than, less than (fewer), most, least)
To match numbers and quantities.
To locate numbers on a number line.
To read & write numbers from 1-20 in numerals and words.
To identify odd and even numbers.
To understand ordinal numbers.
To compare numbers up to 20 (and beyond).
To describe and extend number sequences.
To make ten.
To regroup (carry out a fair swap).
To make ten and count on (in concrete).
To identify ten and count on (in pictorial).
To count out a 2 digit number to 20 and regroup in the 1s.
To partition and recombine numbers to 20 into 10s and 1s (teen numbers).
To partition and recombine any 2 digit number into 10s and 1s.

Mental maths

To count to and across 100
To count larger collections by grouping into tens, then fives or twos.
To count backwards in ones from any two digit number
To count on any given single digit number from any two digit number (count on seven from 22) To count in multiples of 2, 5 and 10
To count on in tens from a tens number stopping at a given number. (count from 20 to 60) To count back in tens from a tens number stopping at a given number (count from 80 back to 30) To describe and extend number sequences: counting on or back in steps of ones or tens from any given number. Count in 2s from 0-20, count in 2s from any given number
To identify one more and one less than any givennumber
Can say whether any number from 1-100 is odd or even and why.
Count in tens from zero… from 40… from 8
Count in 2s from zero, count from 1,3,5
To say what number comes next in a given pattern. (16,14,12, □,□)
To recall number bonds (see addition strand forexemplification)
To know number bonds of all numbers within 10 (6 = 1+5, 5+1, 4+2, 2+4 etc)
To know number bonds to 10, To know number bonds within 20
To make a reasonable estimate (then count to check)
To state the value of the digits in a two digit number (14 is one ten and four ones)

Year 1 – Number and place value Progression (a combination of these models and images can be used for every objective)

To find one more/less than a given number

To compare quantities

'More than' to compare objects.
'There are more green apples than red apples'

'There are fewer/less red apples than green apples'

'Greater than' to compare numbers. 5 5 is greater than 3 **3** 3 is smaller than 5 Count and compare

To match numbers and quantities.

Can you make this amount using unifix cubes?

To locate numbers on a number line.

Where would 15 be on the number line?
How do you know? 10 20 30
On a bead string? On a counting stick?

To read and write numbers to 20

To identify odd and even numbers

Use 'pairs of' to represent even

Represent up to 9 using ten grid.

To understand ordinal numbers

Circle the fourth elephant

To compare numbers up to 20. (fewer/more) (smaller/greater)

To describe and extend number sequences

How many stars in the next pattern?

Find the missing numbers:
15, 14, 13, □, □, □

To make ten

To use bundles of straws for children to feel the 'ten-ness' of ten.

To regroup (carry out a fair swap)

For children to use Dienes to create a 'fair swap' (regrouping of ten ones for one ten)

To make ten and count on (concrete)

Use objects, dienes or bundles of straws to 'make 10' then count on:

11

14

12

To make ten and count on (pictorial)

Draw around ten and then count on.

Ten and eight is

eighteen.

10……. 11,12,13,14,15,16,17,18

To count out a 2 digit number to 20 and regroup in the 1s

To partition and recombine numbers to 20 into 10s and 1s. (teen numbers then beyond 20)

'16 is 1 ten and 6 ones'

whole

part part

Year 2 – Number and place value (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Count in steps of 2,3, and 5 from 0 and in tens from any number, forward and backward Recognise the place value of each digit in a two-digit number (tens, ones)
Identify, represent and estimate numbers using different representations including the number line.
Compare and order numbers from 0 up to 100; use <, > and = signs
Read and write numbers to at least 100 in numerals and in words
Use place value and number facts to solve problems.

Notes and guidance (non statutory)

Using materials and a range of representations, pupils practice counting, reading, writing and comparing numbers to at least 100 and solving a variety of related problems to develop fluency. They count in multiples of three to support their later understanding of a third.
As they become more confident with numbers up to 100, pupils are introduced to larger numbers to develop further their recognition of patterns within the number system and represent them in different ways, including spatial representations. Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction. They
become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder.

Key Vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, regroup, fair swap, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, first, second, third, last, before, after, next, between, half way, place, place value, stands for, represents, round, nearest, estimate. **Key Concepts**

Numbers can be partitioned in many ways into part, part, whole. (Unique partitioning is when numbers are broken up in the standard representation i.e. 63 is 6 tens and 3 ones. Multiple partitioning is the ability to also see: 63 = 5 tens and 13 ones or 2 tens and 43 ones- this is an important tool for mental strategies)
We can regroup ten ones for one ten. Ten tens is one hundred.
In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder. Zero is a place holder and means 'no ones, no tens, no hundreds etc.' Numbers can be compared using the terms'greater than' and 'smaller than' with and without concrete representation.

Potential barriers/misconceptions

Reversal of digits 03 for 30 and 31 for 13. This can create problems when ordering numbers.
Confusion about the place value of numbers. Difficulties especially apparent when ordering numbers such as 212 and 221. Failure to understand that the position of the numeral gives it the value.
Pupils not always sure what makes a 'sensible' answer (not estimating).

Learning objectives (see overleaf for exemplification)

To represent 2 digit numbers (concrete)
To count within 100 by making tens first.
To recognise the place value of each digit in a 2 digit number.
To compare numbers from 0 – 100.
To order numbers from 0-100.
To partition and recombine 2 digit numbers into 10s and 1s.
To partition and recombine 3 digit numbers into 100s, 10s and 1s.
To partition numbers in different ways.
Identify numbers on a number line.
To use the greater than, less than and equals signs (<,>, =)
To begin to round numbers less than 100 to the nearest 10.
Read and write numbers in numerals and words.

Example Questions

Here are some numbers: 44 87 62 28 51. Write them in order; the first one is done for you: 28 □ □ □ □ .
Here are two signs: '< ' ' > '. Use the signs to make the following correct: 54 □16, 19 □ 94, 51□35
Ben puts 12 coins on a table. He hides some of them under his hand. How many coins is Ben hiding?
Tim bought two pieces of fruit. He spent thirty pence altogether. He bought an orange for 12 pence. What did he pay for the other piece of fruit?
Circle two numbers that add to make a multiple of 10: 11 12 13 14 15 16 17 18 19
Fill in the blank to make this correct: 40 – 30 = 10 + □.
Write the two missing numbers in this sequence: □ 45 47 49 51 □ 55 57
Charlie is making 3-digit numbers with these cards. He can make this number: 7 2 4. Write all the other 3-digit numbers he can make. Write the missing digits to make this correct: □ 0 + 3□ = 43
Write an odd number between 34 and 44.
Write the missing numbers in this sequence: 47 42 37 □ □ 22 17 12
Write a number in the space to make this correct: 867 = □+ 60 + 7
Sarah has 60 sweets. She puts 6 sweets in each party bag. How many bags does she put sweets in?

Mental maths

To count in steps of 2, 3, 5 and 10 (forwards and backwards from any given number). To count on in tens from any given number (with and without a hundred square). To count to and across 100 from any given number (forwards and backwards). To have rapid recall of the x2, x3, x5 and x10 tables.
To count up in threes from any given number (forwards and backwards). To find ten more than a multiple of ten (ten more than 40).
To identify 1, 10, or 100 more/less than any given number.
To accurately say the sequence of odd numbers from 1-19.
To say whether any number is odd or even.
To recognise multiples of 10, 5 and 2 and say how they know.
To know the value of each digit (what is the number equivalent to 6 tens and 5 ones?).
To know number bonds within 10 (for number bond exemplification see addition strand). To know number bonds to 10.
To know number bonds within 20.
To use the language of ordinality up to twentieth.

Year 2 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 2 digit numbers (concrete) (for regrouping and making 10 see year 1)	**To count within 100 by counting tens first.**	**To represent numbers as tens and ones in a place value table.**	**To compare numbers**
Count 'bundles' of 10: 5 tens = 50 Make 2 digit numbers with dienes: 4 tens + 4 ones = 44	10, 20, 30, 40, 50, 60…… 61, 62, 63, 64, 65, 66, 67, **68** There are 68 in the group. 60 and 8 make 68.	68= □ tens + □ ones 68 = 60 + 8 **6 8** What are the missing numbers? Can you makethese numbers? 97, 62, 33, 48…	Which number is greatest? How much greater is it? If the tens are equal then we compare the **ones.** 25 is 3 more than 22 22 is 3 less than 25
To order numbers from 0-100	**To partition and recombine 2 digit numbers into 10s and 1s. (unique partitioning)**	**To partition and recombine 3 digit numbers into 100s, 10s and 1s. (unique partitioning)**	**To partition numbers in different ways. (multiple partitioning)**
Tens ones 2 3 2 5 3 3 Abstract: pupils to place range of numbers in order.	57 = 50 + 7	173 = 100 + 70 + 3 173 100 70 3	64 can be partitioned as = 6 tens and 4 ones (60+4) 64 can also be partitioned as: 64 64 or 50 14 40 24
Identify numbers on a number line.	**To use the greater than>, less than< and equals= signs (<,>, =)**	**To begin to round numbers less than 100 to the nearest 10.**	**Read and write numbers in numerals and words. (to 100 and beyond)**
Locate numbers on a number line What number is marked by the arrow? How do you know? Can you locate: 22, 39, 94…..on the number line? On an unmarked number line?	22 ‹ 54	Look at the ones 5 or above? – round on to the next tens number Less than 5? – round back to the previous tens number	Note that when recording numbers over 20: **22** twenty**-**two (this is written with a hyphen between the tens &ones)

Key steps in developing understanding of place value

1) Ten- 1 ten has a different value to 1 one
2) Tens and ones- Tens and ones can exist side by side
3) Number names- Instead of naming a number 1 ten and 4 ones we give it a name-fourteen
4) Hundreds, tens and ones- Children should be aware that when they have ten tens they must 'regroup' to make 1hundred
5) Numbers can be broken into different parts. The number 36 can be 3 tens and 6 ones. It is also 2 tens and 16 ones

Year 3 – Number and place value (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum	Notes and guidance (non- statutory)
Pupils should be taught to : Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number. Recognise the place value of each digit in a three-digit number (hundreds, tens, ones) Compare and order numbers up to 1000 Identify, represent and estimate numbers up to 1000 in numerals and words. Solve number problems and practical problems involving these ideas.	Pupils now use multiples of 2,3,4,5,8,10, 50 and 100 They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146=100+ 40 and 6, 146=130+16) Using a variety of representations, including those related to measure, pupils continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.
Key vocab: count, count(up) to, count on (from, to), count back (from, to), more, less, many, few, odd, even, every other, how many times?, pattern, pair, ones, tens, hundreds, thousands, regroup, digit, equal to, greater, more, larger, bigger, less, fewer, smaller, compare, order, last, before, after, next, between, half way. **Key concepts** Number lines, hundred squares and Dienes all help to understand counting, ordering and place value. In a three digit whole number, if a digit is to represent hundreds it must be written in the place on the left and where necessary the middle and right hand spaces filled with zeros as place holders. Each column to the left of another is ten times greater; each column to the right is ten times smaller. Numbers can be partitioned in many ways into part, part, and whole. (Unique partitioning is when numbers are broken up in the standard representation i.e. 63 is 6 tens and 3 ones. Multiple partitioning is the ability to also see: 63 = 5 tens and 13 ones or 2 tens and 43 ones- this is an important tool for mental strategies) Numbers can be compared using the terms 'greater than' and 'smaller than'.	**Learning objectives** (see overleaf for exemplification) To represent 3 digit numbers (concrete) To find 1, 10 or 100 more than a given number (concrete). To recognise the place value of each digit in a three digit number. To use part, part whole to partition numbers in different ways. To compare numbers up to 1000 To order numbers up to 1000 Identify, represent and estimate numbers up to 1000 in numerals and words. To recognise the place value of differentmeasures. To use dienes and coins to understand placevalue.
Potential barriers/misconceptions Ordering numbers is challenging if pupils don't have a strong understanding of place value. Trouble giving values to each of the digits. In 23 the value of the first number is not 2 it is 2 tens or 20. Confusion around zero as a place holder. Pupils move into abstract too quickly and although they can recognise and read numbers up to 1000 they are unfamiliar with the place value of each digit.	**Mental maths** Rapid recall of 2, 3, 4,5,8,10,50 and 100 tables. To count in multiples of 4 and 8. (x2 table double to get x4, double again to get x8) To count in multiples of 50 and 100. To find 10 or 100 more than a given number. To find 10 or 100 less than a given number. To count on and back in tens crossing hundreds from any given three digit number. Count on in tens from one given number to another. How many tens did you count? (336 to 416) Count back in tens from one given number to another. How many tens did you count? (202 to 182) Count on or back 400 in hundreds from any tens number. (from 500, from 520, from 570) Count back in hundreds round a circle of children, starting with Lucy with 970. Who will say 370? To know the next number in a sequence. (256, 356, 456, 556…) To identify all odd and even numbers up to 1000. To know what odd number comes before/after a given number. After a given number. (What odd number comes before 301?) To know what happens when odd numbers are added together. To know what happens when even numbers are added together. Create sequences with a given constraint. i.e. make a sequence with 107 and 116 in it. To know that multiples of: 100 end in 00

Example Questions

Write these numbers in order of size, starting with the smallest: 903 1094 912 106 190

What number is ten less than four hundred and three?

Fill in the blanks to make this correct: 360 + □ + □ = 540

Look at these digit cards: '6 '' 9 ' ' 7' . Use each card once to make the largest number. Use each card once to make the smallest even number.

Write the missing number in the space: □+ 47 = 100

Circle three numbers that add to make 160: 10 30 50 70 60

Write 'always', 'sometimes or 'never' in each space to make the following sentences correct: Multiples of 2 end in 3. Multiple of 5 end in 5. Multiples of 10 end in 0.

Write a calculation that you could do to check the answer to: 150 + 350 =500

3 5 4 6. Use each of these digits once to make a total that is a multiple of 5. □ + □ =

Write these numbers in order: 164, 146, 106, 160, 140 (from smallest to largest).

Here are the first five numbers in a sequence: 420, 400, 380, 360, and 340. The sequence continues in the same way. Write the number that will be 10th in the sequence.

What number is ten less than 1002?

Write in figures the number one thousand and thirty.

Choose three of these number cards to make an even number that is greater than 400. 3 8 9 1 Write one thousand, three hundred and forty-seven to the nearest ten.

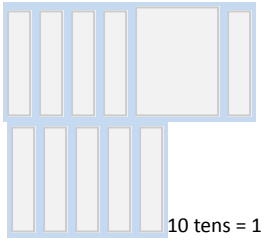
50 end in 00 or 50
10 end in 0
5 end in 5 or 0
2 end in 0, 2, 4, 6, 8,

To know what multiple of 10, 5, 100 etc lies before/ after a given number. (What is the multiple of 5 after 805?) To say what digits represent. (What does the digit 3 represent in 345? = 3 hundreds)

To know which numbers lie between given numbers. (What even numbers lie between 415 and 420?) To round numbers to the nearest 10 or 100.

Year 3 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 3 digit numbers (concrete)



10 tens = 1 hundred

10, 20, 30, 40, 50, 60, 70, 80, 90, **100**.

(ten hundreds = 1 thousand)

= one hundred and three = **103**

= two hundred and twenty-one = **221**

4 1 5

To find 1, 10 or 100 more than a given number (concrete).

Count on one

221 + 1 = one more than 221 is 222

+ 1

221, 222

Count on ten

221 + 10 = ten more than 221 is 231

Count on one hundred

221 + 100 = one hundred more than 221 is 321

+ 100

221, 321

				2 3 1 **+ 10** 221, 231	

To recognise the place value of each digit in a three digit number.	**To use part, part whole to partition numbers in different ways.**	**To compare numbers up to 1000**	**To order numbers up to 1000**
 2 1 5 Stands for : 2 hundreds 1 ten 5 ones or 200 or 10 or 5	146 = 100+40+6 or 146 = 130 + 16 146 or 146 100 40 6 130 16	 338  334 First compare the hundreds, then the tens, then the ones 338 is greater than 334 (334 is smaller than 338)	Arrange these numbers in order. Begin with the smallest. 476, 259, 601 **First compare the hundre ds** 601 is greater than 476 and 259. 476 is greater than 259. **In order from smallest: 259, 476, 601**

Identify, represent and estimate numbers up to 1000 in numerals and words.	**To recognise the place value of different measures.**
Write numbers in words: 999 = nine hundred and ninety-nine 234 = two hundred and thirty-four **500 1000** 303 = three hundred and three What number is here on the number line? How do you know?	139cm = 100 cm + 30 cm + 9 cm (100cm = 1 metre)

	To use Dienes and coins to understand place value
In 506 = 5 hundreds, 0 tens and 6 ones 506 = 500 + 0 + 6 In 506: The digit 6 is in the ones place, the digit 0 is in the tens place and the digit 5 is in the hundreds place.	**The idea of 'regrouping' can be reinforced through the exchanging of coins.**

Year 4 – Number and place value (When planning ensure you track back to year 3 and forwards to year 5)

National curriculum

Count in multiples of 6,7,9,25 and 1000
Find 1000 more or less than a given number
Count backwards through zero to include negative numbers
Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens and ones) Order and compare numbers beyond 1000
Identify, represent and estimate numbers using different representations
Round any number to the nearest 10, 100, 1000
Solve number and practical problems that involve all of the above and with increasingly large positive numbers. Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

Notes and guidance (non-statutory)

Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice.
They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.
They connect estimation and rounding numbers to the use of measuring instruments.
Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.

Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property.

Key concepts

In a two digit whole number the digit indicating the multiple of 10 is written on the left, and that to distinguish between, say 20 and 2, a zero is put in the space on the right as a place holder.
Zero is a place holder and means 'no ones, no tens, no hundreds etc.'
Negative numbers can be found on a temperature scale.
Negative numbers are below zero and the size of the negative number indicates the distance it is from 0. < is less than and > is greater than. When we multiply by 10/100 the place value of the digits change. Rounding numbers can alter a situation (i.e. results of a race etc.) A number line can be used to visualise decimals.

Potential barriers/misconceptions

Ordering numbers is challenging if pupils don't have a strong understanding of place value. Trouble giving values to each of the digits. In 23 the value of the first number is not 2 it is 2 tens or 20.
Confusion around zero as a place holder.
Pupils move into abstract too quickly and although they can recognise and read numbers up to 1000 they are unfamiliar with the place value of each digit.
When multiplying by 10/100 children think they just 'add zero' without understanding that the place value of the digits have changed and zero then becomes a place holder.

Example Questions

The sum of two numbers is 100. Write the missing digits: 3 □ + □ 3 = 100
Each missing digit in this sum is a 9 or a 1. Write in the missing digits. □□ + □□+ □□= 201
Paul says, "Every multiple of 5 ends in 5". Is he correct? Explain how you know.
Write in figures the number five thousand and thirty-two
Jet has these numbers: 1330 1303 1033 1003 1030. She writes them in order from smallest to largest. What is the fourth number she writes?
The temperature in London is 3°C. Paris is 9 degrees colder than London. What is the temperature in Paris? Circle the numbers nearest to 1000. 1050 1340 1046 1004 1040
Match 3500 to numbers with the same value: 35 hundreds 3500 ones 35 tens 350 tens 350 hundreds. Write these prices in order from smallest to largest: 97p £11.50 £0.76 £8 £3.05
Write these amounts in order in the boxes: £60.06 £60.60 £6.60 £6.06
John makes a sequence of numbers. His rule is: "find half the last number, and then add 10". Write the next two numbers in his sequence: 36 28 24 _ _
Circle the number that is about the same as the correct answer to 49 + 48. 10 50 40 100 70 200

Learning objectives (see overleaf for exemplification).

To represent 4 digit numbers (concrete- place value counters).
To find 1, 10, 100 or 1000 more than a given number (concrete).
To recognise the place value of each digit in a four digit number.
Order numbers beyond 1000.
Compare numbers beyond 1000.
Round any number to the nearest 10, 100, 1000. (To round appropriately given context see division strand) To identify and count in negative numbers.
To estimate and round numbers using measuring instruments.
To understand the history of different numeration systems.
To read and understand Roman numerals.
To understand the place value of decimals and fractions (see learning objectives in these strands).

Mental maths

To count in multiples of 6,7 and 9
To count in multiples of 25 and 1000
To count backwards through zero to negative numbers.
To find 1,10, 100, 1000 more than any given number (with 4 or more digits)
To find 1,10, 100, 1000 less than any given number (with 4 or more digits)
To multiply by 10, 100 and 1000 (understanding that digits move to the left when multiplied by 10…). To know what the value of each digit is up to 10,000.
To count on from any given number crossing boundaries (count on 7 in ones from 669, 70 in tens from 669, 700 in hundreds from 669, 7000 in thousands from 2669).
To round any two or three digit number to the nearest 10 or 100.
To round measurements in seconds, minutes, hours, metres, kilometres, litres to the nearest 10 or 100 units. Estimate calculations by approximating. (608+297 = 610+300= approximately 910)
Approximate multiplications (19x16 = 20x16 = (2x16) x10= 320)
Extend and explain number sequences (48, 41, 34, 27…) continuing beyond zero.
To notice a pattern when counting from zero in 2s, 4s then 8s (4s are double 2s, 8s are double 4s) To recognise odd and even numbers up to 10,000 and make general statements about them. (if you add odd numbers the answer is even. Check. Explain why?)

Year 4 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 4 digit numbers (concrete- place value counters)

= 2341 Using place value counters:

= 2341

(Pupils to regroup tens 1s for 1 ten counter etc.)

To find 1, 10, 100 or 1000 more than a given number (concrete).

Count on by tens

2341 + 10 = ten more than 2341 is 2351

+ 10

2341, 2351

Count on by hundreds

2341 + 100 = one hundred more than 2341 is 2441 **+ 100**

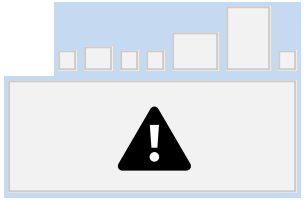
2341, 2441

Count on by thousands

2341 + 1000 = one thousand more than 2341 is 3341 **+ 1000**

2341, 3341

To recognise the place value of each digit in a four digit number.



Stands for : 1 thousand 2 hundred 1 ten 5 ones or 1000 or 200 or 10 or 5

= 1000+200+10+5 = 1215

In 7506 = 7 thousands, 5 hundreds, 0 tens and 6 ones 7506 = 7000+ 500 + 0 + 6

In 7506: The digit 6 is in the ones place, the digit 0 is in the tens place, the digit 5 is in the hundreds place and the digit 7 is in the thousands place.

Compare numbers beyond 1000

Which is greater/smaller?

= 2214

=2421

If the thousands are same compare hundreds.

Order numbers beyond 1000

Arrange these numbers in order. Begin with the smallest.6476, 4259,4601

First compare the thousands

6476 is greater than 4259 & 4601

.

4601 is greater than 4259.

In order from smallest: 4259, 4601, 6476.

Round any number to the nearest 10, 100, 1000

To identify and count in negative numbers.

Negative numbers are numbers that are lessthan zero. Ensure thermometer is shown in both directions

Negative numbers < zero < positive numbers

To estimate and round numbers using measuring instruments

How much water is there? To the nearest 10ml What time is it to the nearest 5 minutes?

How much does this weigh? To the nearest 100g?

How long is this? To the nearest cm?

To understand the history of different numeration systems (Derek Haylock, Mathematics explained for primary teachers, 2006)

To read and understand Roman numerals.

Egyptian Hieroglyphics Roman numerals I I IIIII V ∩ X ᘐᘐᘐᘐ L ⁹ C ⁹⁹ ⁹⁹ ⁹ D ⁹⁹⁹ ᘐᘐᘐᘐ IIIII CCCLXVI and 500. The Hindu Arabic system (used today) uses fe based on the place value system.The system uses ten a to count).	**Hindu-Arabic** 1 5 10 50 100 500 366 c system (3000BC) had separate symbols for ten, hundred, tho dred thousand and a million. The Romans (3000 years later) ba on a similar system including additional symbols for 5, 50 We ased on where it is written i.e. 200, 20 or 2. The Roman syster orII. In our Hindu-Arabic place value system, all numbers can finite set of digits: 0,1,2,3,4,5,6,7,8,9.		 L = 50 C = 100 D= 500 M=1000 MMXV= 2015

Year 5 – Number and place value (When planning ensure you track back to year 4 and forwards to year 6)	
National Curriculum **Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit. Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000. Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero.** **Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.** **Solve number problems and practical problems that involve all of the above.** **Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.**	**Notes and guidance (non-statutory)** **Pupils identify the place value in large whole numbers** **They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions they have met so far.**
Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, divisibility. **Key concepts** If we need to work in the concrete to consolidate our understanding we can use place value counters. We work from left to right when determining the place value. We increase the powers of ten as we move from right to left. 10 thousands = 1 ten thousands When counting in steps of powers of 10, we are multiplying by ten which changes the place value. The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it). The number line and use of ordinal numbers are useful when introducing the concept of negative numbers. To associate positive and negative integers the number line can be shown both horizontally and vertically.	**Learning objectives** (see overleaf for exemplification) To represent 6 digit numbers (to 1 000 000) (concrete- place value counters). To recognise the place value of each digit in a six digit number. To compare & order numbers to at least 1 000 000 To recognise and describe linear number sequences. To find the term-to-term rule To interpret negative numbers To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000 (To round appropriately in context see division strand) To count in steps of powers of 10 up to 1 000 000 Read Roman numerals (See progression year 4) To understand decimals and fractions (see strands on decimals and fractions).

Potential barriers/misconceptions

As the numbers increase, pupils find it difficult to read numbers aloud.
When counting in 1000s pupils sometimes unsure what comes after 9000 (10 000). Also what comes before and after this number (9999, 10 001)
Pupils do not make the link between these numbers and real life contexts. (Could be linked to populations of countries, costs of items etc.).
Pupils counting in powers of ten are often 'ill advisedly' told that they need to 'add a naught'. Children must observe the transformation of numbers as the digits move into new place on the place value grid.
-5 can also be referred to as 'negative 5' as 'minus 5' can suggest the need for subtraction.

Example Questions

□ and Δ each stand for a different number. What is their value? □ = 34 □ + □ = Δ + Δ + □
Here are four digit cards. '7' '5' '2' '1' Choose two cards each time to make the following two-digit numbers. The first one is done for you: An even number - 52. A multiple of 9 - _ _ a square number - _ _ a factor of 96 - _ _ In the number 15083, what does the 5 represent? Thousands, hundreds, tens, ones.
A car costs more than £8400 but less than £9200. Tick the prices than the car could cost: £8397 £9190 £9230 £8999
Round each number to the nearest whole number: 5.01 8.51 6.65
James has 84 stamps. Emily has 57 stamps. Which of these is the BEST way to estimate how many stamps there are altogether: 90 + 60 = 150 80 + 60 = 140 80 + 50 = 130
What does the digit 3 in 305 642 represent?

Mental maths

To count in steps of powers of 10 up to 1 000 000: 10, 100, 1000 etc (see exemplification over page). Count forwards and backwards with positive and negative whole numbers including through zero. Know the value of every digit in six digit+ numbers.
To compare two numbers (which is less 4 thousands or 41 hundreds).
To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2).
To know 1000, 10,000, 100,000 more/less than any six digit number.
To multiply any number by 10, 100 and 1000 (and explain how the place value changes).
To divide any number by 10, 100 and 1000 (and explain how the place value changes).
To multiply decimal numbers by 10, 100 and 1000 (and explain how the place value changes). To divide decimal numbers by 10, 100 and 1000 (and explain how the place value changes). To identify the number that sits halfway between two numbers. (i.e. 27,400 and 28,00)
To place six digit numbers in ascending and descending order.
To look at a quantity (i.e. coins in a jar, grapes in a bowl) and make a reasonable estimate. To round any two, three, four digit number to the nearest 10, 100, 1000.
To round measures. (i.e. distance between cities to the nearest km)
To identify the best approximation.
To calculate the rise and fall in temperature using both positive and negative integers.

Year 5 – Number and place value Progression (a combination of these models and images can be used for every objective)

To represent 5 & 6 digit numbers (to 1 000 000) (concrete- place value counters).

5 2 5 2 2

Fifty-two thousand, five hundred and twenty-two

Children should be able to read numbers when given as digits and write numbers when given as words.

To recognise the place value of each digit in a six digit number.

Six five ten one three one two Hundred thousands thousand hundred ten ones thousand

In 651,312= six hundred and fifty-one thousand, three hundred and twelve.
651,312 = 600,000 + 50,000 + 1000 + 300 + 10 + 2

To compare & order numbers to at least 1 000 000

When comparing numbers look at the value of each of the digits starting from the left.
Arrange these numbers in order. Begin with the smallest.16476, 14259,14601

First compare the Ten thousands
16476 is greater than 14259 & 14601.
14601 is greater than 14259.
In order from smallest: 14259, 14601, 16476.

To round numbers to the nearest 10, 100, 1000, 10 000 and 100 000

We round off numbers so that we are able to estimate. We use the approximation sign ≈ to stand for **approximately equal to**. It shows rounding off of the numbers.

Round off to the nearest 100

What is 9872 rounded off to the nearest hundred?
9872 is between **9800** and **9900**
9872 is nearer to **9900** than **9800**
9872 is **9900** when rounded off to the nearest hundred. 9872 ≈ 9900

Round off to the nearest 100

To recognise and describe linear number sequences.

To find the term-to-term rule

To interpret negative numbers

Use subtraction to identify if sequence is: Increase or decrease **755,482 705,482 655,482 605,482** **70**5,482 is **50,000** less than **75**5,482 **65**5, 482 is **50,000** less than **70**5,482 **60**5,482 is **50,000** less than **65**5, 482 50,000 less than **60**5,482 is **55**0,482 The next number is **550,482**	'To explain rule for a sequence and work out what comes next.'	Familiar contexts such as temperatures, multi-story buildings, heights above and below sea level and bank balances can give meaning to positive and negative integers.	What is 8276 rounded off to the nearest thousand? 8276 is between **8000** and **9000** 8276 is nearer to **8000** than **9000** 8276 is **8000** when rounded off to the nearest thousand. 8276 ≈ 8000
To count in steps of powers of 10 up to 1 000 000			

Larger whole numbers than 9 are constructed using powers of the base: ten, a hundred, a thousand etc.

Name of power	Numerals	Constructed from tens	Expressed as powers of ten in symbols	Expressed as powers of ten in words
A million	1000000	10x10x10x10x10x10 =	10^6	Ten to the power six
A hundred thousand	100000	10x10x10x10x10=	10^5	Ten to the power five
Ten thousand	10000	10x10x10x10x=	10^4	Ten to the power four
A thousand	1000	x10x10x10=	10^3	Ten to the power three
A hundred	100	10x10=	10^2	Ten to the power two
Ten	10	10=	10^1	Ten to the power one

Year 6 – Number and place value (When planning ensure you track back to year 5 for progression)	
National Curriculum **Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit. Round any whole number to a required degree of accuracy Use negative numbers in context, and calculate intervals across zero. Solve number and practical problems that involve all of the above.**	**Notes and guidance (non-statutory)** **Pupils use the whole number system, including saying, reading and writing numbers accurately.**

Key vocab: numeral, place value, order, round, stands for, represents, regroup, >, greater than, <, less than, integer, positive, negative, above, below, zero, minus, next, consecutive, sort, classify, property, factor, factorise, square, prime

Key concepts

The decimal point is used as the separator in the contexts of money and measurement.
When counting in steps of powers of 10, we are multiplying by ten which changes the place value. When rounding numbers we talk about 'rounding on' and 'rounding back' to link to the number line/counting stick rather than rounding up and down.
The context for rounding is the most important element. Are pupils rounding up when buying packets of tiles for the floor (so as not be short of tiles) or are they rounding back to the nearest 5 minutes when catching a train (so as not to miss it)?

Potential barriers/misconceptions

Pupils counting in powers of ten are often 'ill advisedly' told that they need to 'add a naught'. Children must observe the transformation of numbers as the digits move into new place on the place value grid. Some pupils may still write three hundred and forty seven as 30047, showing a lack of understanding around zero acting as a place holder. When looking at the number 300, the position of the three is what makes is 300, rather than the zeros. The function of the zero is to make this position clear and to signify no tens and no ones. Pupils may hear 'tens' and 'hundreds' if the wording is not articulated clearly when saying 'tenths' and 'hundredths'.

Learning objectives (see overleaf for exemplification)

To consolidate learning objectives from year 5
Then:
To understand the place value of 7 digitnumbers
To identify negative integers.
To calculate intervals acrosszero.
To find the term-to-term rule
To order and compare numbers up to 10 000 000
To round any whole number (To round appropriately given context see division strand)
To extend place value to decimals
To identify decimal numbers on a number line

Example Questions

Imagine you have 25 beads. You have to make a three-digit number on an abacus. You must use all 25 beads for each number you make. How many different three-digit numbers can you make? Write them in order.
Here are some digit cards: '2' '4' '6' '6' Write all the three digit numbers, greater than 500, that can be made using these cards.
Tariq makes a sequence of 5 numbers. The first number is 2. The last number is 18. His rule is to add the same amount each time. Write the missing numbers: 2 _ _ _ 18
Sarah is working with whole numbers. She says: "If you add 2 two-digit numbers you cannot get a four-digit number". Is she correct? Explain why.
The temperatures were: Inside: -2°C Outside: - 10°C What is the difference between these two temperatures? The temperature inside an aeroplane is 20°C. The temperature outside is - 30°C. What is the difference between these temperatures?
Round each decimal to the nearest whole number: 5.01 8.51 7.75
Write half a million in figures.
Write 2 and a half million in figures.
Which two of these numbers when multiplied together have the answer closest to 70? 7.4 8.1 9.4
10 Write a decimal which is greater than 0.7 and less than 0.71
Write these numbers in order of size. Starting with the smallest: 1.01 1.001 1.101 0.11
Write down a multiple of 4 that is greater than one thousand.

Mental maths

To count in multiples of any number up to x12 forwards and backwards from any given number. To count in steps of powers of 10 up to 1 000 000 (see exemplification year 5)
To count in 11s, 15s, 19s, 21s, 25s then back. Can you go past zero?
To count in steps of 0.1, 0.5, 0.25 to 10 then back.
To multiply and divide whole numbers by 10, 100, 1000
To multiply and divide decimal numbers by 10, 100 and 1000
Count forwards and backwards with positive and negative whole numbers including through zero. Know the value of every digit in six digit+ numbers.
To compare two numbers (which is less 4 thousands or 41 hundreds?).
To make the biggest/ smallest integer possible with a range of digits (i.e. 8 3 0 7 6 0 2).
To know 1000, 10,000, 100,000 more/less than any six digit number.
To round any whole number to the nearest multiple of 10, 100 or 1000
To put integers in order from smallest to largest crossing zero. (-37, 4, 29, -4, -28)
To make statements about identification of odd and even numbers.
To find all the prime factors of any number to 1000 (the prime factors of 60 are 2,2,3 and 5, since 60= 2x30 = 2x2x15 = 2x2x3x5.)
Use factors for finding products mentally (32x24 = 32 x 3 x 8 = 96 x 8 = 800 – (4 x 8) = 768
Identify numbers with an odd number of factors (squares)
Identify two digit numbers with only two factors (primes)
Recognise prime numbers.

Year 6 – Number and place value Progression (a combination of these models and images can be used for every objective)		
To understand the place value of 7 digit numbers.	**To identify negative integers.**	**To calculate intervals across zero.**

One million, six hundred and forty-nine thousand.

In **1,649,000**
The digit 1 stands for 1,000,000. The value of the digit 1 is one million
The digit 6 stands for 600,000. The value of the digit 6 is six hundred thousand
The digit 4 is in the ten thousands place. Forty thousand. (40,000)
The digit 9 is in the thousands place. Nine thousand (9,000)

Concrete apparatus cannot be used to teach negative integers.

Positive and negative integers are seen as reflections in zero.
+3 to be referred to as 'positive three'.
-3 to be referred to as 'negative three'.

Visualrepresentations can be used to calculate the difference between positive and negative integers.

To find the term-to-term rule

To order and compare numbers up to 10 000 000

When we compare numbers, we look at the value of each digit starting from the left.

9 hundreds is greater than 8 hundreds.
So 1,712,935 is greater than 1,712,846

To round any whole number

We round off numbers so that we are able to **estimate**.
We use the approximation sign ≈ to stand for **approximately equal to**. It shows rounding off of numbers.

Estimate the value of 6327 x 7 (round off the 4 digit number to the nearest thousandfirst)
6327 x 7 ≈ 6000 x 7
= 42000 6400 ÷ 8

Estimate the value of 6742 ÷ 8 (6742 ÷ 8 using knowledge of x8 tables) 7200 ÷ 8
6742 is nearer to 6400 than to 7200. So 6742 ÷ 8 ≈ 6400 ÷ 8 = 800.

To extend place value to decimals

Thousands	Hundreds	Tens	Ones Tenths	Hundredths	Thousandths
1000	100	10	1 0.1	0.01	0.001
10 x 10 x 10	10 x 10	10	1		
10^3	10^2	10^1	10^0 10^{-1}	10^{-2}	10^{-3}

To identify decimal numbers on a number line

1.35 can be explained in the context of length as 1 metre + 3 tenths of a metre + 5 centimetres

On the number line it lies between 1 and 2
It lies between 1.3 and 1.4
It lies between 1.34 and 1.36.

Chapter 2 Addition

EYFS 1 – Addition (When planning ensure you track forwards to year 1)
Early Learning Goal 11 Children count reliably with numbers from 1 to 20 place them in order (see number and place value) Say which number is one more or one less than a given number Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Key vocab: add, more, and, make, sum, total, altogether, score, double, one more, two more, how many more? **Key concepts**

Adding is the combining of a set of objects.
Know that a group of things changes in quantity when something is added (or taken away).
More than means 'added on to'.
The number of objects does not change even if moved around (Conservation).
We know that when counting a group, the last number represents the total quantity.

Potential barriers/ misconceptions

Unable to recite numbers in the correct order.
Not associating number names with objects in group.
Unable to count without putting in line or touching.
Not being able to 'hold' the number they started with when adding the second group.
Not knowing the number order when counting on from any given number.

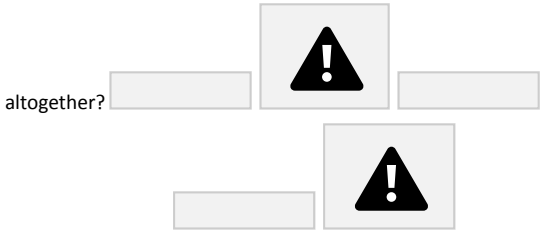
Learning objectives (turn over for exemplification)

To make comparisons between quantities.
To use language of quantities such as 'more' and 'a lot'.
To use the language of more to compare sets of objects.
To separate a group of 3 or 4 objects in different ways (total still same).
To respond to (and use) addition vocabulary in rhymes and games.
To find the total number of items in two groups by counting all of them.
To know that when counting a group the last number represents the total amount. To find one more than a given number.

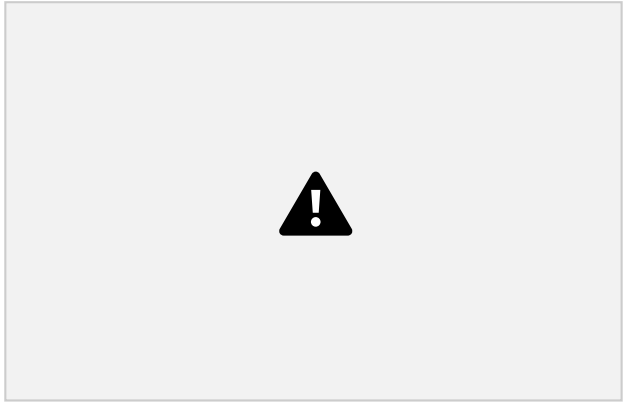
Example Questions

I am going to add one more button to this set of four buttons. How many buttons will there be then?

Show me five fingers using both your hands. Show me another way.
There are four balls in the bag. Put two more balls in the bag. How many balls all together are in the bag? One, two, three... what goes next?
Hop two spaces on this number track. Now hop three more. Where are you now?
I have two carrots in a bowl. I add two more carrots to the bowl. How many carrots are in the bowl now? Find all the dominoes that have a total of five spots.
Show me three fingers on one hand. Show me five fingers on the other hand. How many fingers altogether? I have hidden two cups in this box. There are three cups on the table. How many cups are there altogether? How many different ways can we put five sweets on two plates?
How many yellow pencils are there? How many red pencils are there? How many pencils are there altogether?



Mental Maths (can revisited throughout day once concept has explicitly shared)



EYFS 1 – Addition Progression (a combination of these models and images can be used for every objective)

To make comparisons between quantities	To use language of quantities such as 'more' and 'a lot'.	To use the language of 'more' to compare sets of objects.	To separate a group of 3 or 4 objects in different ways (total still same).

Which group of sweets would you like? Why? 	Please may I have some more milk?  I have a lot of conkers. 	Isaac has more blocks than me. 	How many different ways can we put four teddies in two beds? 
To respond to (and use) addition vocabulary in rhymes and games	**To find the total number of items in two groups by counting all of them.**	**To know that when counting a group the last number represents the total amount.**	**To find one more than a given number**
	 Three paper plates can be used to represent part, part whole. Children move the cars together into one group to find the total amount. (starting with 0-5)		One more than three is four 

EYFS 2 – Addition (When planning ensure you track forwards to year 1)

Early Learning Goal 11
Children count reliably with numbers from 1 to 20, place them in order
Say which number is one more or one less than a given number
Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

KS1 ready:
Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Represent and use number bonds and related subtraction facts within 10
Add and subtract one-digit and two-digit numbers to 20 including zero
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations , and missing number problems such as 7 = ☐ - 9

Key vocab: add, more, and, make, sum, total, altogether, score, double, one more, two more, how many more? **Key concepts**

Adding is the combining of a set of objects
Adding is the combining of two numbers
When we add numbers we can break them into 'bits' to make it easier. Part, part, whole: 5 + 1 = 6 Know that a group of things changes in quantity when something is added (or taken away).
We count on from the bigger number.
More than means 'added on to'
The number of objects does not change even if moved around (Conservation)
We know that when counting a group, the last number represents the total quantity. (After = sign)

Potential barriers/misconceptions

Unable to recite numbers in the correct order.
Not associating number names with objects in group
Unable to count without putting in line or touching.
Not being able to 'hold' the number they started with when adding the second group.
Not knowing the number order when counting on from any given number.

Example Questions

Flora has 4 stickers and Romeo has 2 stickers. How many do they have altogether? (combining 2 groups) Sidney has three stickers and he is given two more. How many stickers does he have now? (Augmenting) There are five children in the home corner. One more joins. How many are there now?
Jordine has no carrots on her plate. Marvin puts four carrots on her plate. How many carrots does Jordine have now? What number is one more than five (use the objects to help).
Take away one egg from the set of seven below. How many are there now?

[Make a set of cards with jackets on. Arrange them randomly on a table.] Find two jackets that have four buttons altogether. Are there any other possibilities?
[Put four balls into a bag and then close it. Show two more balls in your hand]. How many balls are there altogether?

Learning objectives (turn over for exemplification)

To say the number that is one more than a given number.
To combine two or more quantities to find the total (combining) To find one more than a group of up to five, then ten objects.
Uses quantities and objects to add two single-digit numbers and count on to find the answer: To count on when adding to a group. (holding first number in head)
To add two sets of objects that are the same (cars+ cars)
To add two sets of objects that are different (apples + bananas)
Increase one quantity by a given amount to find the total (augmentation)
Records using marks they can interpret and explain
To recognise and name + and = signs
To read an addition number sentence
To solve an addition number sentence
To arrange an addition number sentence
To know doubles to 10.
To know number bonds to 5, 6 and 10

Mental Maths (can revisited throughout day once concept has explicitly shared)

EYFS 2 – Addition Progression (a combination of these models and images can be used for everyobjective)

To say the number that is one more than a given number.	Combine two or more quantities to find the total (combining)	To find one more than a group of up to five, then ten objects.	To count on when adding to a group. (holding first number in head)

Maria has five sweets and Ismail has 3 sweets. How many sweets do they have altogether? (grouped)

One more than five is six

To add two sets of objects that are the same (cars+ cars)

Increase one quantity by a given amount to find the total (augmentation) (where one quantity is increased by some amount)

Records using marks they can interpret and explain

To recognise and name + and = signs

To add two sets of objects that are different (apples + bananas)

Maria has five sweets and she is given 3 more.
How many does she have in total? (increase)

Children count 5 into bag. (hidden)
Then count on from 5. (6,7,**8**)

There are 8 sweets in the bag.

Where is the addition sign?
+
How do we know to add?
+
Where is the total?
=
What does this sign mean?

=

To read an addition number sentence

To solve an addition number sentence

To arrange an addition number sentence

To know doubles to ten

To know number bonds to 5, 6 and 10

To read aloud:
4 + 3 = 7 Knowing that + is add = is equals

To use objects to lay out & solve: **4 + 3 = 7**

To match number cards to objects to make number sentence:

1 + 1 = 2 4 + 4 = 8 2 + 2 = 4
5 + 5 = 10 3 + 3 = 6

Part, part whole: number bonds to

5.

Progression towards bar model

Children start by adding objects to a group

What is two more than 4?

Children can then use unifix cubes, counting on from the greater number, to find the total number of

cubes.

Year 1 – Addition (When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Represent and use number bonds and related subtraction facts within 20.
Add and subtract one-digit and two-digit numbers to 20 including zero
Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = - 9

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9+7=16; 16-7=9, 7=16-9)
They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations. Pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, more than and less than so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Key vocab: add, more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, how many more to make…?, how many more is… than…? How much more is…?

Key concepts

Relate addition to counting on
Adding two or more numbers gives another number.
A group of things changes in quantity when something is added.
When adding a 1 digit number to a 2 digit number- make ten i.e. 8+4 = 8+2+2= 12
When I add two objects to a group, and then take two objects away the total is unchanged.

Potential barriers/misconceptions

- Learn the pairs of numbers to 10 and 20 but not the pairs that total each number up to20.
- Don't understand the commutativity of 3+7 = 7+ 3
- Don't associate number facts e.g 13+4=17 and 17-4=13 as they don't see + and – asinverse. - Only able to complete empty box questions when on right hand side (answer) rather than any position. 3+ =8 -When counting on from a given number, include the start number in their counting. (6+3 = 6,7,8 = 8) rather than 6+3= 7,8,9 = 9)
- count on and back in 10s and 1s not combining i.e. when add 9, add 10 and subtract 1.(adjust)

Learning objectives (turn over for exemplification)

To add with number bonds within 10
To know all number bonds to 10
To add with number bonds to 20
To investigate all possible sets of two numbers to make a given number.
To partition numbers into part, part, whole.
To use a number line to count on.
To use inverse (write corresponding subtraction facts to given addition facts- number families). To add two 1 digit numbers using 'make 10' strategy
To add a 1 digit and a 2 digit number using the 'regrouping into tens and ones' strategy. To solve one step word problems using 'part-whole' or adding on concept.

Example Questions

What numbers go in the boxes: 12 + 5 =□ 12+ □ =15
□ + 5 =17 □ +□ = 15
I think of a number. I add 5 the answer is 9. What's my number?
What is 18 plus 5? What is the sum of 16 and 8? What is 15 more than 4?
How many different ways can I put 8 fish into 2 fish bowls?
On a number line I show 7 + 5. I start at 7 and do 5 jumps. What if I started at 5? How many jumps would I need to do? Would it be the same answer? Why?
How can you use the part, part whole model to make 12? How many ways can you make 12?

Mental Maths

To add multiples of ten
To add ten to any two digit number by counting in 10s
Bridge through ten (and 20 etc) when adding a single digit number. (Making ten). i.e. 8 + 6 = 8 + 2 + 4 = 14
Count on from the largest number
Rapid recall of number bonds
Use of near doubles to add (6 + 7 = 6 + 6+ 1 = 13)
Add 9 to a single digit number by adding 10 and subtracting 1 (adjust)

Year 1 – Addition Progression (a combination of these models and images can be used for every objective)

To add with number bonds within 10

To know all number bonds to 10

To use the inverse

To add with number bonds to 20

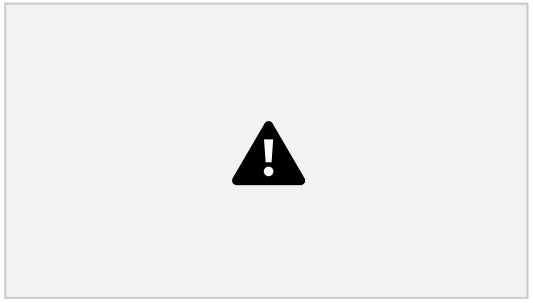
To investigate all possible sets of two numbers to make a given number

Solve missing number problems

To use a number line to count on.

To add a 1 digit number using 'make 10' strategy (bridging through multiples of 10)

When performing addition within 20 children move towards number conservation. The total is unchanged when objects/counters are moved from the second frame to 'make ten'. Knowledge of number bonds to 10 is important. (this involves breaking the smaller number into 2 parts)



To add a 1 digit and a 2 digit number using the 'regrouping into tens and ones'strategy.

Pashur has made 16 muffins. His sister has made 3. How many altogether?

Regroup 16 into 10 and ones.

1)Regroup into tens and ones
2)Add the ones: 6+3 =9
3)Add the ten: 10 + 9 = 19 the same as 16 + 3 = 19
There are 19 muffins altogether

Vertical addition with no regrouping. (2 digit + 1 digit not bridging 10)

Using a place value chart & Dienes.
22 + 6 =

20 + 8 = 28 **2 8**

Progression towards bar model

Year 2 – Addition (When planning ensure you track back to Year 1 and forwards to year 3)

National Curriculum

Solve problems with addition and subtraction: -Using concrete and pictorial representations, including those involving numbers, quantities and measures. -Applying their increasing knowledge of mental and written methods.

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:- A two digit number and ones

- A two digit number and tens

- Add two two-digit numbers

- Adding three one digit numbers

Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practice addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3+7=10; 10-7=3 and 7=10-3 to calculate 30+70=100; 100-70=30 and 70=100-30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. 5+2+1=1+5+2= 1+2+5) This establishes commutativity and associativity of addition.

Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Key vocab: add, addition more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…?

Key concepts

Adding can be done in any order.
Counting on from the biggest number is not always most useful way: 8 + 5 + 5 (make 10 first). To add 2 digit numbers we partition.
We add the ones first and then the tens.

Potential barriers/misconceptions

Pupils believe they have to add in the order that the question was asked (not understanding that addition can be done in any order to do mental calculations more efficiently).
Still don't have secure rapid recall of addition facts. i.e. struggle to identify all possible missing numbers in _ + _ = 7. (Number bonds).
Makes mistakes counting teen numbers or crossing boundaries.
Is insecure in making links between addition and subtraction and/or recognising inverse.
In vertical addition- placing the answer in the wrong column. i.e. 24 as 2 in the ones, 4 in tens.

Learning objectives (turn over for exemplification)

To use the counting on strategy (with number line, Dienes or mentally) To use making ten strategy to add (see y1 progression)
To use partitioning to add
To add a two digit number and tens
To add a two digit number and ones without regrouping
To add 2 two-digit numbers without regrouping
To regroup and rename
To add three one-digit numbers
To add numbers with regrouping (in ones)
To add numbers with regrouping (in tens.)
Use the inverse to solve missing number problems
To solve one step word problems using 'part, whole' and adding on.

Example Questions

Addition questions phrased in a variety of ways:
64+ 10 Add 60 to 17 24 plus 36
What is the sum/ total of 18 and 7?
How many are 5 and 19 altogether?
Which two/three numbers could have a sum of 12?
What must I add to 26 to make 30?
I think of a number. I add 20. My answer is 50. What is my number?
Andre has 37 football cards. He buys 30 more. How many does he have now?

Mental Maths

Counting forwards/ backwards from any given number
Rapid recall of all addition facts to 20 & 100
Partitioning adding the ones and then the tens: 24+13 = 4+3+ 20+10=37
'Make ten' adding three one digit numbers: 6+7 = 6+4+3= 13
Compensating 24+9= 24+10-**1**=33 or 42+21= 42+20+**1**= 63 (adjust)
Near doubles: 30+29 = double 30-1 and 14+15 is double 14 + 1 or double 15-1

Year 2 – Addition Progression (a combination of these models and images can be used for every objective)

To use the counting on strategy

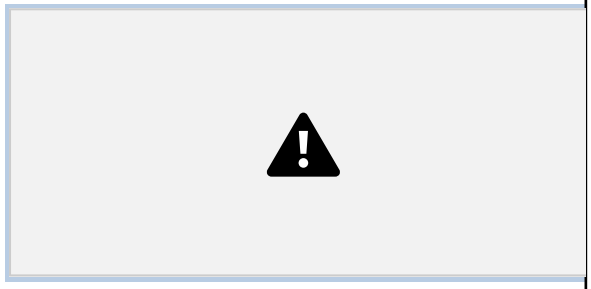
To make ten also shown in y1 progression

To partition

To add a two digit number & ten

Add tens using:
Hundred square (pattern)
Jumps of ten on a number line
Counting stick add ten from any given number 23, 33, 43 etc.
Add 'ten' Dienes each time.

To add a two digit number and ones without regrouping. (not bridging 10) also shown in y1 progression

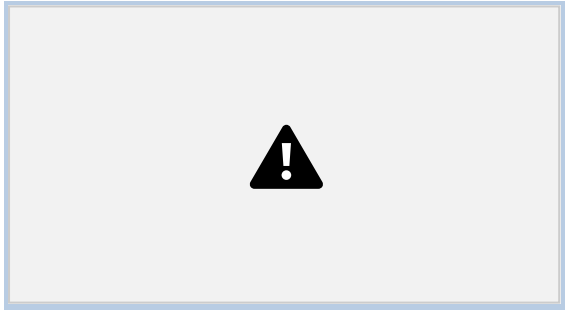


To add 2 two-digit numbers without regrouping (not bridging 10)

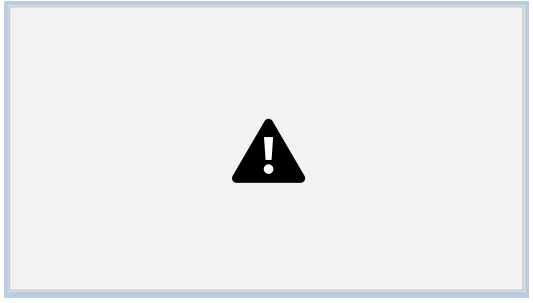
To regroup and rename

To add three one-digit numbers

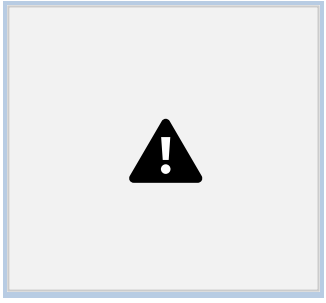
To add numbers regrouping in ones (bridging 10) (Expanded method)



To add numbers regrouping in tens. (bridging 100) (compact method)



Use the inverse to solve missing number problems



To solve one step word problems:

Using the bar model: comparing (addition)

Year 3 – Addition (When planning ensure you track back to year 2 and forwards to year 4)

National Curriculum : Add and subtract numbers mentally, including:

- **A three-digit number and ones**
- **A three- digit number and tens**
- **A three- digit number and hundreds**

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Estimate the answer to a calculation and use inverse operations to check answers. Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Notes and guidance (non-statutory)

Pupils practise solving varied addition a subtraction questions. For mental calculations with two digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning and practice using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.

Key vocab: add, addition more, plus, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary.

Key concepts

Addition can be done in any order
Addition is the same as: more, add, sum, total, altogether
Addition reverses subtraction (addition is the inverse of subtraction)
It is best to start adding from right to left. Add the ones, then the tens etc
We can move from the expanded method to the compact method for addition.

Potential barriers/misconceptions

Children may still not be secure with all addition facts for each number to 20.
Confused that addition is associative- 3+1 = 4 and 1+3=4.
Find it challenging to mentally add using 'near multiples of 10'. Not sure about which way to compensate: 26+19= 26 +20 – 1 often confused as 36 + 20 + 1.
Sometimes begin adding with the left hand column first
Not understand the concept of regrouping when the number totals more than ten, hundred etc. Children find it difficult to add when there is a zero involved.
Children don't understand importance of zero as a placeholder.

Learning objectives (turn over for exemplification)

To use partitioning to add
To use a number line for addition
To solve missing number problems
To add a three digit number and ones without regrouping (see progression year2)
To add a three digit number and tens without regrouping (see progression year2)
To add 2 three-digit numbers without regrouping
To add three-digit numbers with regrouping (revert to expanded method if tricky)
To add using place value counters
To develop and recognise patterns in addition
To estimate the answer to a calculation
To solve word problems

Mental Maths

Rapid recall of all addition facts up to and including 20
Derive quickly addition doubles from 1+1 to 20+20 e.g. 19+19=38
Doubles of multiples of 5 from 5+5 to 100+100 e.g. 95+95 = 190
Derive quickly pairs of multiples of 5 that total 100: e.g. 65 + 35
Know by heart all multiples of 100 that total 1000: e.g. 400 + 600 = 1000
Add several numbers by: making ten & adjusting when adding 11 or 9 add 10 and +1/-1. Partition and recombine: e.g. 24 + 35 = 20 + 30 + 4 + 5 = 59

Example Questions *Addition questions phrased in a variety of ways:* 194 add 10, Add 60 to 280, 70 plus 50. What is the sum/total of 26 and 39? How many altogether are 121 and 345? Increase 431 by 22. Which two numbers could have a total of 102? Which three numbers? There are 25 people standing on the bus and 62 sitting down. How many people in total? Ali has 298 football stickers. He collects another 121. How many does he have now? Adam has read 173 pages. He has 62 left to read. How many pages in total?	Identify the corresponding subtraction facts. e.g. 22+57 = 79 and 79-57=22 etc. Add a two-digit number to a multiple of 100.e.g. 200+64 Add a two-digit number to a multiple of 10 crossing 100. e.g. 80 + 34 = 114 Add 10 to any number crossing the hundreds boundary. e.g. 196 + 10 Add a pair of multiples of 10, crossing 100. e.g. 90 + □ = 130 Add pairs of multiples of 100 crossing 1000. e.g. 500 + 800 Add 100 to any 3 digit number, without crossing 1000. e.g. 347 + 100 =

Year 3 – Addition Progression (a combination of these models and images can be used for every objective)

To use partitioning to add	To use a number line for addition		To solve missing number problems
To add 2 three-digit numbers without regrouping (see year 2 progression for further examples)	**To add three-digit numbers with regrouping**	**Progression of regrouping:**	**To add using place value counters (moving on from Dienes)**
		Addition with no regrouping (not crossing, 10, 100 etc) Addition regrouping in ones (crossing ten) Addition regrouping in tens (crossing 100) Addition regrouping in tens and ones.	

To develop and recognise patterns in addition	To estimate the answer to a calculation	To solve one step word problems

		
Using the bar model: Combining sets of objects:	**Aggregation: joining one or more sets to another set:**	**Comparing two sets:**
		

Year 4 – Addition (When planning ensure you track back to Year 3 and forwards to year 5)

National Curriculum

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two step word problems in context, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency.

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions withdecimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
We can use Dienes or place value counters to help solve addition problems.

Addition is:

Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of quantity with another: one quantity has a certain amount more than the other. We can use a number line to round numbers to the nearest 10 or 100.

Learning objectives (turn over for exemplification)

To add four digit numbers (no regrouping)
To add with regrouping in the 100s
To add with regrouping in the 100s, 10s and1s
To add with regrouping in the 1000s, 100s, 10s and 1s
To identify common misconceptions in columnaddition
To round off numbers to the nearest 10 / 100.
To estimate to check answers
To add decimals up to 2 decimal places
To solve two step word problems.

Potential barriers/misconceptions

Children sometimes begin adding with the left handcolumn first. **Th H T O** Pupils line up numbers from left to right rather than right to left. i.e. 3056+ 254: **3 0 5 6** **2 5 4**

Not understanding the concept of 'regroup' when a number totals more than ten, hundred etc As numbers get larger pupils miscalculate because of lack of understanding of place value.

Some pupils will not remember to add the ten/hundred that they have regrouped.

Pupils don't use estimation skills to predict answer.

Lack of understanding around value of decimal numbers.

Forgetting to include or line up decimal point.

Example Questions

3964 add 30 add 500 to 9544

Which three numbers could have a total of 350?

□ + 88 = 120 7.6 + 5.8 = □

There are 654 girls. There are 276 more boys than girls. How many children altogether?

John, William and Oliver are saving their money to buy a computer game. John has £25. William has £10 more than John. Oliver has the same amount as John and William together. How much do they have?

Mental Maths

Rapid recall of all addition facts to 20. (e.g. all pairs of numbers to 15)

Derive quickly related facts: e.g. 9+6=15, 90+60=150, 900+600=1500

Derive quickly number pairs that make 100. 34 + □ = 100 , □ + 45=100

Derive pairs of multiples of 50 that total 1000: e.g. 250+750

Derive quickly addition doubles from: 1+1 to 50+50 e.g. Double 46

Multiples of 10 from 10+10 to 500+500: e.g. double 280

Multiples of 100 from 100+100 to 5000+5000: e.g. double 17000

Count on from any given number in repeated steps of 1,10,100,1000

Partition into hundreds, tens and ones to add mentally

Add or subtract the nearest multiple of 10, 100 or 1000 and adjust: add 9, 19, 29 or 11, 21, 31 to any number. e.g. 48+ 61 = 48+60+1

Identify addition and subtraction facts for any given algorithm.

Add three numbers mentally. (two digit and one digit)

Add three digit multiples of 10: e.g. 430+360 or 570+260

Find what to add to a three digit number to make the next higher multiple of 100. e.g. 246+ □ = 300 Add numbers to 1 decimal place to make the next whole number. 3.4 +□ = 4.0

Year 4 – Addition Progression (a combination of these models and images can be used for every objective)

To add four digit numbers (no regrouping)

To move from concrete representation or place value tables to support into:

4371 + 5208

	Th	H	T	Ones
	4	**3**	**7**	**1**
+	**5**	**2**	**0**	**8**
	9	**5**	**7**	**9**

Also find the sum of 4 digit + 3 digit.

To add with regrouping in the 100s

(for use of Dienes see year 3)

To add with regrouping in the 100s, 10s and 1s (Same pattern for regrouping in 1000s etc.)

To identify common misconceptions in column addition

Pupils look at examples and identify mistakes.

Children articulate steps in procedure. **2266**

+ 1956

1)Add 6 ones and 6 ones to get 12 ones.

2)Regroup 12 ones into 1 ten and 2 ones

3)Add 6 tens, 5 tens and 1 ten to get 12 tens.

4)Regroup 12 tens into 1 hundred and 2 tens

5) Add 2 hundreds, 9 hundreds and 1 hundred to get 12 hundreds.

6) Regroup to get 1 thousand and two hundreds.

7) Add 2 thousands 1 thousands & 1 thousand to get 4 thousands

To round off numbers to the nearest

To estimate to check answers

To add decimals up to 2 decimal places

10 / 100.		
	Round numbers to the nearest 10 to check reasonable answers: 128 + 254 = 130 + 250= **380** (Actual answer = 382)	
Using the bar model (to solve two step word problems)		**One step word problem with decimals:**
		Mark drank 0.75 litres of orange juice and 1.25 litres of water. How many litres did he drink altogether? (*Bar model to visualise and column method to solve).* 

Year 5 – Addition (When planning ensure you track back to Year 4 and forwards to year 6)	
National Curriculum **Add and subtract whole numbers with more than 4 digits, including formal written methods (columnar addition and subtraction)** **Add and subtract numbers mentally with increasingly large numbers. Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.** **Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.**	**Notes and guidance (non-statutory)** **Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency** **They practise mental calculations with increasingly large numbers to aid fluency (e.g. 12,462+ 4200= 16,662)**

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is… than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions withdecimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
We can use Dienes or place value counters to help solve addition problems.

Addition is:

Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of a quantity with another. i.e. one quantity has a certain amount more than the other. We can use a number line to round numbers to the nearest 10 or 100.

Learning objectives (turn over for exemplification)
(for exemplification also look at year 4 progression)

To add four digit numbers **(**regrouping in the 1000s, 100s, 10s and 1s)
To identify common misconceptions in column addition
To round off numbers to the nearest 10.
To round off numbers to the nearest 100.
To add decimals up to 2 decimal places
To add money using the column method (regrouping)
To add measures using the column method (regrouping)
To find the missing value
To use part, part whole to add money
To solve two step word problems using the bar model.

Potential barriers/misconceptions

As numbers get larger, pupils miscalculate due to lack of understanding of place value. Some pupils will not realise that they need to add the regrouped number.
Pupils sometimes forget to line up the decimal points when adding using the column method- particularly when adding mixed amounts. e.g. £4.50 + 72p

Example Questions

-Nadia is working with whole numbers. She says 'if you add a two digit number to a two digit number you cannot get a four digit number.' Is she correct? Explain why.
-□ and ○ each stand for a different number. If □ = 34 then what is the value of ○**?**
□ + □ = ○ + ○ + □
What is the sum/total of 753 and 227? How many altogether are 854 and 622?
Increase 250 by 420.
Find all the different totals you can make by using three of these 5 numbers:
14721, 76, 9534, 788, 6, 1.07, 0.3, 37.03, 17.73, 31.7

Mental Maths

Add four digit multiples of 100 e.g. 3700 + 4500
Add three or more digit multiples of 100 e.g. 400 + 800 + 500
Add a single-digit multiple of 100 to a three or four-digit number crossing 1000 e.g. 300 + 876 =□ 300 + □ = 1176 □+ 876 = 1176
& 638 + 500=
Add a three digit multiple of 10 to a three digit number without crossing the hundreds boundary. e.g. 230+364 460+518
Find what to add to a three digit number to make the next higher multiple of 100. E.g. 651 + □ = 700 Find what to add to a decimal with units and tenths to make the next higher whole number e.g. 8.25 + □ = 9.0

Year 5 – Addition Progression (a combination of these models and images can be used for every objective)

To add four digit numbers (regrouping in the 1000s, 100s, 10s and 1s)	To identify common misconceptions in column addition

To add decimals up to 2 decimal places

To round off numbers to the nearest 10

To round off numbers to the nearest 100

To estimate: **1296 + 2508**
Children encouraged to articulate their thinking:
'3800 is my estimate because I rounded 1296 to 1300 and 2508 to 2500. 1300 +2500 = 3800'
(To check I did 1300 + 2508 = 3808 – 4 = 3804)

To add money using the column method (regrouping)

To add measures using the column method (regrouping)

1.72 metres + 750 cm + 1500 cm

= 3970 cm or 39.7 m

1720cm
750cm
+ 1500cm
3970cm
1

(**convert so all the same unit of measurement)**

To find the missing value.

3 4 □ 2 1 .☐7 + 1 3 2 9 + 0 .
9 1 4 7 9 1 2 .6 8 1 1

To use part, part whole to add money.

Using the bar model (to solve two step word problems)

Most effective approach would also show step 2 in a bar model. What would this look like?

Year 6 – Addition (When planning ensure you track back to year 5 for progression)

National Curriculum

Perform mental calculations, including with mixed operations and large numbers. Use their knowledge of the order of operations to carry out calculations involving the four operations

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Solve problems involving addition, subtraction, multiplication and division.

Use estimation to check answers to calculations and determine, the in context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction etc. and use the formal written methods of columnar addition and subtraction.

They undertake mental calculations with increasingly large numbers and more complex calculations

Pupils round numbers to a specified degree of accuracy, for example to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example 2+1 x 3 = 5 and (2 + 1) x 3 = 9

Key vocab: add, addition more, plus, increase, make, sum, total, altogether, score, double, near double, one more, two more, ten more, one hundred more, how many more to make…?, how many more is…than…? How much more is…? Tens boundary, hundreds boundary, inverse.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions withdecimals.
Estimating can be used to predict the answer and the inverse can be used to check it. Addition is:
Combining two or more quantities into one
The enlargement of a quantity, i.e. increasing the amount in the quantity.
Comparison of a quantity with another. i.e. one quantity has a certain amount more than the other.

Learning objectives (turn over for exemplification)

To solve any additions with numbers to 2 decimal places. (see y5 progression)
To carry out calculations involving the four operations.
To work systematically to solve a problem
To solve multi step word problems.
To use estimation to check answers to calculations.
To add negative numbers.
To understand the order of operations using brackets.

Potential barriers/misconceptions

Unless a pupil has a good understanding of place value they will continue to make mistakes with column addition. Such errors are often dismissed as common mistakes, when the pupil in fact has a fundamental weakness in their understanding. When adding the decimals such details are highlighted with the positioning of the decimal point.
Students get mixed up with operation and signs when there are subtraction and negative signs in a problem. i.e. important to refer to operation as subtract/ add and these signs as positive/negative.

Example Questions

Add 4250, 3536 and 242.
Which three numbers could have a total of 1? Are there any others?
□ + 4.47 = 6.38 91 + □ + 38 = 250
Find the mean of the following set of numbers….
What totals are possible with these three dice?
Is there a pattern when you add two consecutive numbers?

Mental Maths

(building on Mental Maths from y5)

Find the difference by counting up through the next multiple of 10, 100 or 1000: 7000-3675 is +5 + 20 + 300 + 3000= 3325
Identify near doubles: 421 + 387 = 808 (double 400 plus 21 minus 13)
Add or subtract the nearest multiple of 10, 100 or 1000 adjust: add 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 etc by adding 1,2,3 and adjusting by 0.1.
Add or subtract four digit multiples of 100
Find what to add to a decimal with units, 10^{th} and 100ths to make the next higher whole number or 10^{th}. What must be added to 7.78 to make 8?
Add or subtract a pair of decimal fractions each less than 1 and with up to 2 decimal places.

Year 6 – Addition Progression (a combination of these models and images can be used for every objective)

To work systematically to solve a problem

There are 20 cars and bicycles in a car park. The total number of wheels is 50. How many bicycles are there?

Cars Bikes No. wheels 50 wheels?

10 10 40+20=60 Too many 9 11 36+22=58 Too many 8 12 32+24 = 56 Too many 5 **15** 20 + 30 = 50 Yes

Use one set of digit tiles in the following task:

0 1 2 3 4 5 6 7 8 9

The sum of two three digit numbers is a 4 digit number. (No digit used more than once).

□ □ □

+ □ □ □

□ □ □ □

For a problem like this students need to know addition but also have a good number sense and use 'guess and check'.

Each symbol stands for a different digit.

□ Δ What would each digit stand for?

□ Δ What can you say about the digit ¤?

+ □ Δ What can you say about the digit □? ○ ¤ What can you say about the digit ○?

Try to work systematically, trying out different possible values for Δ.

To add negative numbers.

To add negative numbers.

To solve multi step word problems (using the bar model)

Most effective approach would also show step 2 in a bar model. What would this look like?

Chapter 3

Subtraction

EYFS 1 – Subtraction (When planning ensure you track forwards to EYFS 2 & year 1)
Early Learning Goal 11 **Children count reliably with numbers from 1 to 20, place them in order (see number and place value)** **And say which number is one more or one less than a given number** **Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.**

Key Vocab: take (away), leave, how many are left/left over? , how many have gone?, one less, two less, how many fewer is… than…?, difference between, is the same as.

Key concepts

Subtraction can be experienced by generating experiences that include change. Cakes eaten, balloons popped.

Subtraction is the taking away of one amount from another.

Concept of subtraction needs to be taught and experienced through play as they are the precursors to subtraction reasoning.

Potential barriers

Unable to recite numbers in the correct order

Not associating number names with objects in group

Unable to count without putting in line or touching.

Not being able to 'hold' the number they started with when taking away from the group. Not knowing the number order when counting backwards.

Learning objectives (see overleaf for exemplification)

To make comparisons between quantities.
To use the language of fewer (less) to compare sets of objects.
To separate a group of 3 or 4 objects in different ways (total still same).
To know that a group of things changes in quantity when something is taken away.
To respond to (and use) subtraction vocabulary in rhymes and games.
To find the total number of items after some are taken away by counting all of them.
To know that when counting a group the last number represents the quantity.

Example Questions

Using a play house- put three people in one room and four in another.
'Which room has more people in?'
'How do you know?'
Move some people from one room to another- 'What has happened in this room?'

Which plate has fewer biscuits on?
Ellie has three apples, Diane has two apples. Who has fewer apples? Ellie or Diane? (use apples to show). I am going to take away one of these five cubes. How many will be left?

Mental strategies (can revisited throughout day once concept has explicitly shared)

Join in rhymes and sing songs such as:
Five little ducks went swimming one day
Five little speckled frogs
Five little monkeys jumping on the bed
Five current buns in the baker's shop
Alice the camel has ten humps
Ten green bottles
One man went to mow...

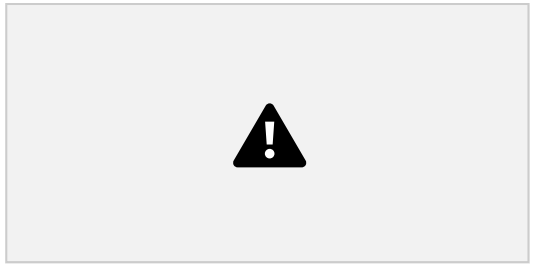
Say the number name that goes before a given number.
Count forwards and backwards using a counting stick

EYFS 1 – Subtraction Progression (a combination of these models and images can be used for every objective)

To make comparisons between quantities.	**To use the language of fewer (less) to compare sets of objects.**	**To separate a group of 3 or 4 objects in different ways (total still same).**	**To know that a group of things changes in quantity when something is taken away.**

Which group of cars would you like to play with? Why?

I have fewer sweets than Bob.



To respond to (and use) subtraction vocabulary in rhymes and games.

Five little ducks went swimming one day

Five little speckled frogs

Five little monkeys jumping on the bed

Five current buns in the baker's shop

Alice the camel has ten humps

Ten green bottles

One man went to mow...

To find the total number of items after some are taken away by counting all of them.

One, two, three, four, five…

To know that when counting a group the last number represents the quantity.

EYFS 2 – Subtraction

(When planning ensure you track back to EYFS 1 and forwards to year 1)

Early Learning Goal 11

Children count reliably with numbers from 1 to 20, place them in order (see number and place value) and say which number is one more or one less than a given number

Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

KS1 ready-

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs. Represent and use number bonds and related subtraction facts within 10

Add and subtract one-digit and two-digit numbers to 20 including zero

Solve one step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as 7 = ? - 9

Key Vocab: take (away), leave, how many are left/left over? , how many have gone?, one less, two less, how many fewer is… than…?, difference between, is the same as.

Key concepts

Subtraction is the taking away of one amount from another.
Concept of subtraction needs to be taught and experienced through play as they are the precursors to subtraction reasoning.
Concrete apparatus should be used prior to experimenting with recording.
The last number in a number sentence/ number after the = sign is the total (not the answer). The subtraction situation of change is often referred to as 'take away'.
To find the answer you need to count how many are left.

Potential barriers

Children unable to relate subtraction to taking away. Teach other phrases for 'taking away' e.g. 'How many less?'
Misunderstanding of 'one less'; do not consistently identify the number before a given number.
Children are unable to bridge from 10 to 11 and 20 to 21 as always 'stop' at 10 and 20 when counting.

Learning objectives (see overleaf for exemplification)

Relates subtraction to taking away.
To find one **less** than a group of up to five, then ten objects.
Records using marks they can interpret and explain.
Uses quantities and objects to subtract two single-digit numbers and count on to find the answer. To count backwards on a number line or counting stick
To recognise and name - and = signs
To read a subtraction number sentence
To solve a subtraction number sentence
To arrange a subtraction number sentence
To halve (an even group up to 12)
To know number families to 5, 6 and 10
To begin to identify own mathematical problems based on own interests and fascinations.

Example Questions

There are five birds in the nest. One flew off. How many are there now?
Look at this group of counters (eight counters positioned randomly) Now look at this group of cubes (five cubes positioned randomly). Are there fewer cubes or fewer counters? How do you know? Choose two number cards (from 1-5) Which of your two numbers is worth more? Which number is less? There are nine biscuits on this plate. Take three of the biscuits to eat. How many biscuits are left on the plate? We have four aprons. There are seven children who want to paint. How many more aprons do we need? John has four books. Lisa has one. How many more books does John have than Lisa?

Mental strategies (can revisited throughout day once concept has explicitly shared)

Say the number name that goes before a given number (one less)
Choose two groups of objects to make a given total. Six blocks. Four red, two green.
Say how many are left when some are taken away by **counting how many are left**. We ate 2 of our six cakes. How many cakes are left? (Count 1, 2, 3, 4, 5, 6. Take away 1, 2… 1,2,3,4 left. Say together: 6 take away 2 is 4)
Say how many are left when some are taken away, by **counting back from a number**. We made 6 mince pies. We ate 2 of them. How many pies are left? (Count back 2 from 6: 5, 4. Say together six take away two is four.)
Find out how many have been removed by **counting up** to the larger number. There were 8 books on this shelf. There are only 5 books now. How many have gone? (Count up from 5 to 8. 6, 7, 8… and say 3. Say together: 5 add 3 is 8. 8 take away 3 is 5).

EYFS 2 – Subtraction Progression (a combination of these models and images can be used for every objective)

Relates subtraction to taking away.	To find one less than a group of up to five, then ten objects.	Records using marks they can interpret and explain.	Uses quantities and objects to subtract two single-digit numbers and count on to find the answer.
	One less than six is five		 5 – 3 = 2 4 – 3 = 1
To count backwards on a number line or counting stick	**To recognise and name - and = signs**	**To read a subtraction number sentence**	**To solve a subtraction number sentence**

	Where is the take away (subtraction) sign? - How do we know to take away? + Where is the total? = What does this sign mean?	To read aloud: **7 - 3 = 4** Knowing that - is subtract/ take away = is equals	To use objects to lay out & solve: **7 - 2 = 5**
To arrange a subtraction number sentence	**To halve (an even group up to 12)**	**To know number bonds to 5, 6 and 10**	**To begin to identify own mathematical problems based on own interests and fascinations.**
To match number cards to objects to make number sentence: **5 - 3 = 2**	 Half of 8 is 4.	Part, part whole: number bonds to 5. whole whole whole 5 5 5 0 5 1 4 2 3 Part part part part	

Progression towards bar model

Children start by subtracting objects from a group **What is one less than 4?**	**Children then use unifix cubes, counting back from the greater number, to find the total number of cubes.** 4 -1 = 3 3 1

Key vocab: - , subtract, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, how many fewer is…than…?, how much less is…? Difference between, half, halve.

Key concepts

When you subtract zero the total stays the same. (related to same concept of add zero)
Understand subtraction as 'take away' and 'find a difference' by counting up.
Subtraction is also used for 'how many more to make?' (complementary addition)
That the related vocabulary for subtraction is: take away, subtract, how many are left, how much less is…than…, difference between, how much more is…than…, how many more to make…
Number bonds help make the connection between addition and subtraction.
Subtraction is associated with the part- whole and the taking away concept.
A family of number sentences can be written from a set of three related numbers.
Two digit numbers can be regrouped into tens and ones.

Potential barriers

Lack of confidence in numbers bonds within ten, to ten and to twenty will prohibit children from fully understanding the rules of commutativity.
Children are confident with counting 'up' but have limited experience counting backwards from any given number. Don't associate number facts e.g 13+4=17 and 17-4=13 as they don't see + and –as inverse.

Doesn't link the language of take away and find the difference.

Example Questions

Can we find the difference between two numbers by counting?
Using a number line show me two numbers that have a difference of two. How might you write that? Which number comes before/after 17? Does 16 always come before 17?
How many are left?
What is the difference between these sets/numbers?
How can we subtract these things/numbers?
There are eight dogs in this kennel and 3 run out. How many dogs are left now?
There are 4 dogs in this kennel and 9 dogs in the other kennel. What is the difference between them? Inverse: There are 3 cats on this chair. 2 more cats jump onto the chair. Now there are 5 cats. If 2 cats jump off, how many will be left on the chair?
If 14+5=19 what else do you know about these numbers?
How do you know you need to take away? What clues are there?
How many different ways can you show me that 12 subtract 4 is 8?
Can you make up a take away question and show me how to do it?
Which numbers in the sequence are missing? Explain how you know: ?, 9, ?, 11, 12

Learning objectives (see overleaf for exemplification)

To break numbers into parts
To subtract with number bonds
To subtract by taking away.
To subtract by counting on.
To subtract small numbers where sets are hidden. (counting on)
To subtract by counting backwards
To subtract within 20 by regrouping into tens and ones
To use a number line to count back.
To make a family of number sentences

To use inverse (write corresponding subtraction facts to given addition facts- number families). To solve missing number problems
To solve one step word problems using part whole method

Mental strategies

Counting stick: counting forwards and backwards in steps (not only of ones) from any given number. 7-3 = count back in ones from 7
15-3= count back in ones from 15
18-6= count back in twos from 18

To use 'count back from' strategies. (8-6= 7,6,5,4,3,2… =2)
To use 'count back to' strategies. (8-6 = 7,6 = 2)
Find a small difference by counting up. (When two numbers are close together i.e. 15-12=3 counting up from 12 to 15 gives 3.)
Subtract ten from a teens number: 19-10= □ 19-□= 9 □-10= 9
Subtract ten from any two digit number, without crossing 100: 49-10 =□ ; 49 - □ = 10 ; □-10 = 39 Subtract a pair of multiples of ten without crossing 100: 50–20=□ ; 50 - □ = 30 ; □ – 20 = 30

Year 1 – Subtraction Progression (When planning ensure you track back to EYFS and forwards to year 2)

To break numbers into parts	**To subtract with number bonds**	**To subtract by taking away**	**To subtract by counting on** **To subtract small numbers where sets are hidden. (counting on)**

5 - 2 = 3

Three and two is five

Five subtract three is two

Five subtract two is three

7 9 is 7 and 2

9 So 9 – 7 = 2 And 9 – 2 = 7

2

How else can we break 9?

7 – 2 = ?

2 …… 3, 4, 5, 6, 7

Counting up strategy using finger or counters.

To use a number line to count back.

To subtract by counting backwards

To subtract within 20 by regrouping into tens and ones

17 – 3 = 14

17 = 10 and 7

7 – 3 = 4

17 – 3 = 14

There are 14 bears left.

To find the difference by counting on

To make a family of number sentences

6

10

4

To solve missing number problems

- and = signs and missing numbers:

(can cover part, part, whole model to visualise) -

9- 7 =□ +

9

9 - □ = 7

2 = 9 - □ -

7 = 9 - Δ

To solve one step word problems using part whole method

How many trees are left standing?

2 part

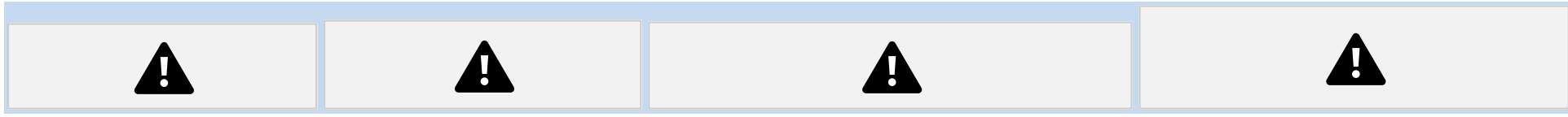
whole

6

4 part

6 – 2 = 4

Progression towards bar model



Year 2 – Subtraction (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Solve problems with addition and subtraction:
-Using concrete and pictorial representations, including those involving numbers, quantities and measures. -Applying their increasing knowledge of mental and written methods.
Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - A two digit number and ones
- A two digit number and tens
- Add two two-digit numbers
- Adding three one digit numbers
Show that the addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference. Pupils practice addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3+7=10; 10-7=3 and 7=10-3 to calculate 30+70=100; 100-70=30 and 70=100-30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (e.g. 5+2+1=1+5+2= 1+2+5) This establishes commutativity and associativity of addition. Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, regroup.

Key concepts

Addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
Subtraction is taking away from a whole.
When we take away we can take away in bits. We subtract the units first, then the ten, then the hundreds. A 2 digit number can be conceptualised as tens and ones.
Number lines can be used to support subtraction by counting backwards.
Number lines can be used for find the difference by counting on.
When using a hundred square to subtract 10, you can 'move up' on the grid.
Subtraction reverses addition (subtraction is the inverse of addition).

Learning objectives (see overleaf for exemplification)

To break numbers into parts
To use the number bond strategy to subtract
To subtract a one digit number from a two digit number without regrouping
To subtract 2 two-digit numbers without regrouping
To use the counting backwards strategy to subtract
Use the 'take away' strategy to subtract
To subtract a one digit number from a two digit number with regrouping
To subtract 2 two-digit numbers with regrouping
To solve one step word problems using 'part, whole'

Potential barriers

Avoid telling children 'you can't take a big number away from a smaller number' you can- this will then go into negative numbers. This could lead to misconceptions at a later point.
Children may not understand the commutative law and believe that it is possible to change any addition and subtraction around.
Children sometimes regroup but see the new number as one and not ten.
Pupils may struggle to see 'find the difference' as a form of subtraction. This can be linked to lack of consolidated skills in counting on and back.
Challenge in recalling addition and subtraction facts to 20
Difficulty using mental strategies to add and subtract two digit numbers.

Mental strategies

To know by heart all addition and subtraction facts for each number to 20
To use number bonds for mental subtraction. 9-4= □ (Think of addition: 4 and 5 make 9 therefor 9-4=5) To subtract multiples of ten from any two digit number
To add and subtract mentally a 'near multiple of ten' to or from a two digit number. (15+39 = 1+39+10+4=54) To find pairs of numbers with a difference of 10, a difference of 9 etc…
To find a small difference when counting up. 84-78 = 79,80,81,82,83,84 = 6
To mentally subtract 11 or 21 or 9 or 19 from any two digit number. 70-11=59 as it is the same as 70-10-1= 59 24-9=15 because it's the same as 24-10+1 = 15
Add or subtract any single digit from any two digit number without crossing the tens boundary (86-□=82) Subtract multiples of ten without crossing 100. (90-40=□)
Subtract multiples of 100 without crossing 1000 (700 – 300 = □)
Use number bonds to find a small difference between a pair of numbers lying either side of a multiple of 10 (102- 97 = 2+3 = 5)

Example Questions

Rapid recall of subtraction asked in a range of ways: 7 take away 3; Take 30 away from 70; 14 subtract 2; Subtract 30 from 70; 3 less than 7; What number must I take from 20 to leave 3?; What is the difference between 10 and 18? How many more is 11 than 3? How many less is 7 than 18?
When Matilda is 4 years old, Sadie is 9. When Matilda is 8, how old will Sadie be?
14 children are on a bus. 8 children get off the bus. 5 more children get off the bus. How many are left on bus?

Year 2 – Subtraction Progression (When planning ensure you track back to year 1 and forwards to year 3)

To break numbers into parts

12 = 5 + 7

12- 7 = 5

How else could we break 12?

To use the number bond strategy to subtract

27 – 4 =

27 - 4

7 – 4 = 3

20 7 then add back on the 20:

20 + 3 = 23

So 27 – 4 = 23

To subtract a one digit number from a two digit number without regrouping (although shown visually as column for progression this could be solved in a number of ways)

27 – 4= 23

First subtract the ones

2 7 27

- 4 - 4

3 23

Seven ones subtract four ones = threeones

Then subtract the tens= 2 tens – 0 tens = 2tens

(check by counting back 4 from 27)

2 tens and 3 ones = **23**

To subtract 2 two-digit numbers without regrouping

To use the counting backwards strategy to subtract

Use the 'take away' strategy to subtract

To subtract a one digit number from a two digit number with regrouping

32 – 9 =

We can't subtract 9 ones from the 2 ones…

So we regroup the tens and ones in 32.

Regroup the tens in 32

3 tens = 2 tens and 10 ones T O

^{2} ^{1}2

First subtract the ones: - 9

3

12 ones – 9 ones = 3 ones

^{2} ^{1}2

Then subtract the tens. - 9

2 3 (2 tens – 0 tens = 2 tens)

So 32 – 9 = 23

To subtract 2 two-digit numbers with regrouping

40 – 29

First we subtract the ones- but we can't!

We regroup the tens and ones in 40

Regroup the tens in 40

4 tens = 3 tens and 10 ones

First subtract the ones: ^{3}4 ^{1}0

- 2 9

1

10 ones subtract 9 ones = 1 one

Then subtract the tens: ^{3}4 ^{1}0

To solve one step word problems using 'part, whole' and the bar model

So, 40 – 29 = 11 - 2 9 3 tens – 2 tens = 1 ten 1 1	Ali has 80 points. Michael has 67 points. What is the difference?

Year 3 –Subtraction (When planning ensure you track back to year 2 and forwards to year4)

National Curriculum : Add and subtract numbers mentally, including:
- **A three-digit number and ones**
- **A three- digit number and tens**
- **A three- digit number and hundreds**

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction Estimate the answer to a calculation and use inverse operations to check answers.
Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Notes and guidance (non-statutory)
Pupils practise solving varied addition a subtraction questions.
For mental calculations with two-digit numbers, the answers could exceed 100.
Pupils use their understanding of place value and partitioning and practice using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, hundreds boundary, regroup.

Key concepts

Subtraction is the inverse of addition
Children must move through the concrete, pictorial then onto the abstract (CPA) in order to fully develop understanding. (dienes and place value discs can be used to support this).
Regrouping can be used in subtraction
When using the column method we subtract from right to left
10 ones = 1 ten
The bar model can be used to find the 'whole' from two or more parts.

Potential barriers

Children sometimes begin subtracting with the left hand column first
In tens and units and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. 945-237 = 712
When the teacher uses the phrase 'near multiple of ten' for mental strategies children often get confused with needing to use multiplication as the operation.

Learning objectives (see overleaf for exemplification)

To find the difference using a number line (for near numbers)
To use number bonds to subtract mentally (see mental strategies below for progression and next page for exemplification)
To subtract without regrouping (see year 2)
To subtract with regrouping in tens and ones
To subtract a 3 digit number with regrouping in hundreds and tens
To subtract a 3 digit number with regrouping in hundreds, tens and ones
To count back to find the difference
To estimate the answer to a calculation
To use inverse operations to check answers
To subtract 'taking away' one set using the bar model
To subtract 'comparing two sets' using the bar model

Mental strategies (All calculations must also use missing number problems: □)

Use number bonds to mentally subtract a 1-digit number from:
-a 2-digit number within 100 with or without regrouping. (ten as the middle stage: 62-7 = 62-2-5= 60-5 = 55) -a 3 digit number within 1000 with or without regrouping in tens and ones
- tens from a 3 digit number within 1000 with or without regrouping in hundreds into tens
- Hundreds from a 3 digit number without regrouping.

Subtract a single digit from a multiple of 100. (600-7=593) (600-□=593)
Subtract a pair of multiples of 10, crossing 100. (120-30= 90) (□ – 30 =90)
Subtract a multiple of 10 from a 2 digit number crossing 100 (112-30=82) (112-□=82)
Subtract a pair of multiples of 100 crossing 1000 (1500-800= 700) (1500-□=700)
Subtract 100 from any 3 digit number, without crossing 1000 (809-100= 709) (□-100=709)
Consolidate subtracting a single digit from a 'teens' number, crossing 10 (use two steps and cross ten as the middle stage: 15-8 = 7 I know this because 15-5-3 = 10-3= 7)
Find pairs of numbers with a difference of 29, 16…
Find the difference between two numbers that are close together by counting up. (504-498 = 2+4=6)

Example Questions 15 take away 8, take 8 from 15, 63 subtract 40, subtract 8 from 15, subtract 40 from 95, ten less than 43, 110 less than 437, what must I take away from 14 to leave 6? What is the difference between ? and ?, 20 taken from a number is 35 what is the number? Using only the numbers 15, 17, 32, 34, 49 write as many different number sentences as you can. What is 100 subtract 24? Subtract 21 from 100. John is 109cm tall. William is 136cm tall. How much taller is William than John? Navneet had £10.00 she spent £2.45. How much money did she have left? There are 265 children at Finching school. 103 have packed lunch, 26 go home for lunch. The rest have school dinners. How many children have school dinners? Show how you worked this out. Chris had 50 books. He sold some and then had 20 left. Which of these is a number sentence that shows this?: □-20=50, 20 -□=50, □50=20, 50-□=20 There are 1000 pieces in a puzzle. 13 go missing. How many pieces are left? Calculate 309-198= There were 24 biscuits in a box. There are now only 18 left. How many have been eaten?	(1003-992= 992+8+3=1003 = 11) Mentally subtract 9,19,29… or 11,21,31 from any two digit number without crossing 100 Develop and recognise a pattern such as 68-5=63, 68-15=53, 68-25=43 therefor 68-45=23 Say the subtraction fact corresponding to a given addition fact: 56+27=83 therefor 83-27=56

Year 3 – Subtraction Progression (When planning ensure you track back to year 2 and forwards to year 4)

To find the difference using a number line (for near numbers)

To use number bonds to subtract mentally

37 - 4

37 R

Regroup the 37 into tens and ones = 3 tens and 7 ones First subtract the ones: 7 – 4 = 3

30 7

Then, add the result to the tens: 30 + 3 = 33

37 – 4 = 33

To subtract with regrouping
(for further exemplification of regrouping see year 2) To move on to subtraction of 3 digit number from a 3 digit number regrouping.

Regroup

regrouped as 6 tens and 12 ones 60 + 12

7 2

- 40 + 7

20 + 5

To count back to find the difference	To estimate the answer to a calculation	To solve missing numbers
74 – 47 = 27	Round numbers to the nearest 10 to check reasonable answers: 58 -- 31 = 60 - 30= **30** (Actual answer = 27)	Can you find the missing numbers in this sequence?

To subtract 'taking away' one set using the bar model	To subtract 'comparing two sets' using the bar model
Dan has £98 that he has been savi ng. He spent £28 on a new game. How much does he have now? 98 – 28 = 70 Dan has £30 now	A mobile phone costs £172 in Phone Shoppaz and £125 with phones.com. How much cheaper is phones.com? **£ 125 £?** phones.com Phone Shoppaz £172 -£ 125= **£47 cheaper at phone.com**

Year 4 – Subtraction (When planning ensure you track back to year 3 and forwards to year 5)

National Curriculum

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

Estimate and use inverse operations to check answers to a calculation

Solve addition and subtraction two step word problems in context, deciding which operations and methods to use & why.

Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency. (build on year 3 mental strategies)

Key vocab: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.

Key concepts

Subtraction as: taking away, finding the difference between and complementary addition. Subtraction is not commutative. i.e. 5-7 is not the same as 7-5

Subtracting a (positive) number makes a number less.

Subtracting zero leaves a number unchanged.

We need to regroup when carrying out a subtraction that crosses a tens, hundreds or thousands boundary.

We can use place value counters to support our conceptual understanding of subtraction It is important to estimate first.

Subtraction should be carried out in a problem solving context.

Learning objectives (see overleaf for exemplification)

To subtract up to 4 digit numbers (no regrouping)
To subtract with regrouping in hundreds and thousands
To subtract with regrouping in hundreds, thousands, tens and ones
To subtract with numbers that have zeros
To identify common misconceptions in column subtraction
To round off numbers to the nearest 10 / 100.
To estimate and use the inverse to check
To subtract decimals up to 2 decimal places
To solve subtraction two step word problems
Use take away and comparing models to solve subtraction word problems.

Potential barriers

When using the column method pupils sometimes begin subtracting with the left hand column first.

In tens and units and other formal vertical calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or the smaller number. e.g. 945-237 = 712

Children may have been incorrectly told 'you can't take a big number away from a small number'. This will cause misconceptions when children start to work in negative numbers.

Pupils don't use estimation skills to predict answer.

Lack of understanding around value of decimal numbers.

Forgetting to include or line up decimal point.

Example Questions

Respond rapidly to oral or written questions explaining the strategy used: 93 take away 7, take 7 from 62, 63 subtract 46, subtract 120 from 215, 170 less than 250, 1000 less than 5437, what must I take from 84 to leave 26? What is the difference between 28 and 65? How many more than 234 is 249? How many less than 68 is 42? What must I add to 54 to make 93? Decrease 72 by 34. 28 add a number is 43, What is the number? Find pairs of numbers with a difference of 79.

1258- 576 = □, 1258- □= 682, Δ - □ = 682

Find the missing number in 91-□=48

Find all the different differences you can make by using two of these five numbers: 219, 193, 74, 156, 97 These are the prices in a shoe shop: Boots = £45.50, Sandals = £12.75 and trainers= £34.99 How much more do the boots cost than the trainers? Rosie buys a pair of trainers and a pair of sandals. How much change does she get from £50?

Mental strategies

Consolidate knowing by heart all addition and subtraction facts to 20. E.g all the pairs for 15: 10+5=15, 5+10=15, 9+6=15, 6+9=15, 8+7=15, 7+8=15 and 15-5=10, 15-10=5, 15-6=9, 15-9=6, 15-7=8, 15-8=7

Know how many steps are taken forwards (+) or backwards (-) when moving on a numberline. i.e. To get from 18 back to 6.

Derive quickly related facts: 160-90=70 therefor 1600-900=700 (1.6-0.9=0.7)

Find the difference by counting up through the next multiple of 10, 100 or 1000. i.e. count from smaller to larger number i.e. 483-386

Count back in repeated steps of 1, 10, 100, 1000 from any given number. i.e. 2003-8=1995 (counting back in 1s from 2003) or 387-50=337 (counting back in 10s from 387)

Partition into hundreds tens and ones: 98-43 = 98-40-3= 55

Subtract the nearest multiple of 10, 100 or 1000 and adjust. i.e. 9, 19, 29 or 11, 21, 31 etc (84-19= 65 because 84-20+1=65) (128-67=61 because it is 128-70+3=58+3=61)

Use the relationship between addition and subtraction (If I know 36+19=55 then I also know: 19+36=55, 55-36=19, 55- 19=36).

Work out mentally one fact: (91-25=□) and then state the other three related facts.

Subtract 2 digit multiples of 10 (130-50=□)

Subtract a pair of multiples of 100, crossing 1000 (□-600=900)

Subtract a multiple of ten from a 2 or 3 digit number without crossing hundreds (76-□=36)

Subtract a single digit from a multiple of 10 or 100 (4000-3=□ or □-3=4997)

Subtract a single digit from a 3 or 4 digit number crossing tens (7003-6899=□ or 5952-□=5949)

Find a small difference between a pair of numbers lying either side of a multiple of 1000 (7003-6988=15 by counting up 2 from 6988 to 6990 then 10 to 7000, then 3 to 7003).

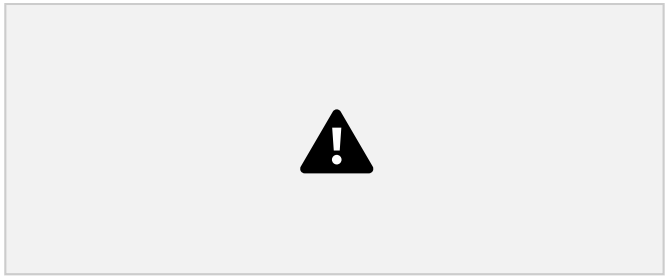
Year 4 – Subtraction Progression (When planning ensure you track back to year 3 and forwards to year 5)

To subtract up to 4 digit numbers (no regrouping)

563 – 241 = 322

To subtract with regrouping (see year 3 for use of concrete)
To identify common misconceptions in column subtraction

563 – 278 = 285



To subtract with numbers that have zero
(although shown visually in the column this could also be solved using the number line)

2000-257

9 9

$^{1}2\ ^{1}0\ ^{1}0\ ^{1}0$

- 2 5 7

1 7 4 3

2000 is rerouped by exchanging 1 thousand for 1 hundreds, 1 hundred for ten tens and ten for ten ones using concrete representation.

To round off numbers to the nearest 10 / 100.

78 to nearest 10

70 75 80

235 to nearest 100

200 250 300

To estimate and use the inverse to check

-Estimate the position of a point on a line.

0 328 -Use rounding to estimate answer. 328 - 189 = 330 - 200= 130 (actual answer: 139)

To subtract decimals up to 2 decimal places

1.5 – 0.7 = 15 tenths – 7 tenths = 8 tenths = 0.8

To solve two step word problems

Nana and Pat were selling tickets for the Hackney festival.
Nana sold 3450 tickets and Pat sold 1286 tickets fewer than Nana.

How many tickets did Pat sell?
How many tickets did they sell altogether?

3450 – 1286 = 2164 Pat sold 2164 tickets 3450 + 2164 = 5614 Nana and Pat sold 5614 festival tickets altogether.

Year 5 – Subtraction (When planning ensure you track back to year 4 and forwards to year 6)

National Curriculum

Add and subtract whole numbers with more than 4 digits, including formal written methods (columnar addition and subtraction)

Add and subtract numbers mentally with increasingly large numbers.

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency

They practise mental calculations with increasingly large numbers to aid fluency (e.g. 12,462+ 4200= 16,662)

Key vocab: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.

Key concepts

Subtraction as: taking away, finding the difference between and complementary addition. Subtraction is not commutative. i.e. 5-7 is not the same as 7-5

Subtracting a (positive) number makes a number less.

Subtracting zero leaves a number unchanged.

We need to regroup when carrying out a subtraction that crosses a tens, hundreds or thousands boundary. We can use place value counters to support our conceptual understanding of subtraction It is important to estimate first.

Subtraction should be carried out in a problem solving context.

Learning objectives (see overleaf for exemplification)

(for exemplification also look at year 4 progression)

To subtract four digit+ numbers **(**regrouping in the 1000s, 100s, 10s and 1s)

To identify common misconceptions in column subtraction

To round off numbers to the nearest 10.

To round off numbers to the nearest 100. (see exemplification year 4)

To subtract decimals up to 2 decimal places

To subtract money using the column method (regrouping)

To subtract measures using the column method (regrouping)

To find the missing value

To solve two step word problems using the bar model.

Potential barriers

Misconceptions can occur when decomposing from a 'high' number. e.g. 9000 -
3654

Some pupils will attempt subtraction calculations using the formal written method, failing to recognise that it would be more efficient to calculate the answer mentally.

Misconceptions occur when pupils (and teachers) use inaccurate language e.g. 2367-
1265

When talking about 2000 – 1000 they may refer to this as 2-1, unaware of the place value of each number. Children can often misplace the decimal point when subtracting decimal numbers.

Example Questions

Respond rapidly to oral or written questions explaining the strategy used. For example: 127 take away 35, take 80 from 373, 678-105, subtract 50 from 225, 500 less than 720. What must I take from 220 to leave 55? What is the difference between 155 and 390? How many more than 952 is 1050? How many less than 305 is 94? What must I add to 720 to make 908? Decrease 92 by 78. 570 add a number is 620. What is the number? Find pairs of numbers with a difference of 599.

□-62=189 7.6-5.8=□ □-256=424 □-Δ= 1.2

141.36-32.58=□

Find the missing number in: 931-□=746

Tilda has read the first 85 pages in a book that is 125 pages long. Which number sentence could Tilda use to find the number of pages she must read to finish the book: 150+85 =□, □-85=150, 150-85=□ Scarves cost £7.95 and hats cost £4.50, £6.50 and £3.99. Chris buys one of the scarves and the £4.50 hat. How much change does he get from £20? Emily buys 2 scarves and a hat. What is the most she could pay?

Mental strategies

Derive quickly related facts such as: 150-80=70, 1500-800=700 and 1.5-0.8=0.7

Find a difference by counting up through the next multiple of 10,100 or 1000 (8006-2993=□ count up from the smaller to the larger number)

Subtract the nearest multiple of 10, 100 or 1000 and adjust (4005-1997= 2008 because it is 4005-2000+3=2008) Recognise that knowing a fact such as 136+319=455 makes it possible to find 455-318 and 455-137 Work out mentally one fact such as 101-25 and be able to state the three other facts in the number family Given the numbers 135, 228 and 363 say or write the four different sentences relating to these numbers Subtract multiples of 10 and 100 (620-380=□ and 6200-3800=□)

Subtract a single digit multiple of 100 from a four digit number crossing 1000 (1263-400=□)

Subtract a three digit multiple of 10 from a three digit number without crossing the hundreds boundary (742-210=□, 742-□= 532, □-210=532)

To find what to add to a three digit number to make the next higher multiple of 100. (651+□=700) Find what to add to a decimal with units, tenths and hundredths (5.71+ □ = 7)

Find the difference between a pair of numbers lying either side of a multiple of 1000 (8004-□=19) Subtract a pair of decimal fractions each less than 1 and with up to two decimal places (0.7-0.26)

Year 5 – Subtraction Progression (a combination of these models and images can be used for every objective)

To subtract four digit+ numbers (regrouping in the 1000s, 100s, 10s and 1s) & to identify common misconceptions in column addition

4249 – 1926 = 2323

4249 1926 ?

When we subtract 1926 from 4249 we get 2323

To subtract decimals up to 2 decimal places

To subtract money and measures using the column method (regrouping)

0.38 – 5.5
£5.50 – 38 p

We can write 5.5 as 5.50

Regroup when 8 hundredths cannot be subtracted from 0 hundredths.

$$\begin{array}{r} 5.\,{}^{4}5\,{}^{1}0 \\ -\,0.38 \\ \hline 5.12 \end{array}$$

Remember to align the decimal point!

5.5 – 0.38 = £5.12

To find the missing value

Use the inverse to find the value in the box:

$$\begin{array}{r} \square\,000 \\ -\,2643 \\ \hline 3357 \end{array}$$

3357 +2643 = 6000 (the missing value is 6)

To solve two step word problems using the bar model.

A piece of cloth 4m long is cut into two pieces. 1.25m

The first piece is 1.25m long
How much longer is the second piece of cloth?

?

4m

2.75m

4m – 1.25m = 2.75m
The second length is 2.75m

2.75m – 1.25 m = 1.5m
The second piece is 1.5m longer than the first piece.

Year 6 – Subtraction (When planning ensure you track back to year 5 for progression)

National Curriculum

Perform mental calculations, including with mixed operations and large numbers.
Use their knowledge of the order of operations to carry out calculations involving the four operations
Solve addition and subtraction multi-step problemsin contexts, deciding which operations and methods to use and why.
Solve problems involving addition, subtraction, multiplication and division.
Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction etc. and use the formal written methods of columnar addition and subtraction. They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils round numbers to a specified degree of accuracy, for example to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
Pupils explore the order of operations using brackets; for example 2+1 x 3 = 5 and (2 + 1) x 3 = 9

Key vocab: - , subtract, subtractions, take (away), minus, leave, how many are left/left over? , how many have gone?, one less, two less, ten less, one hundred less, how many fewer is…than…?, how much less is…? Difference between, half, halve, tens boundary, hundreds boundary, regroup.

Key concepts

We can use addition facts that we know to solve other additions.
We can use addition facts we know to solve additions withdecimals.
Estimating can be used to predict the answer and the inverse can be used to check it.
Key vocabulary: take away, subtract, how many are left, how much left, difference between, how much more, how many more to make, decrease, inverse and the minus sign.
Subtraction as: taking away, finding the difference between and complementary addition.

Potential barriers

Pupils without a strong foundation in place value will continue to make mistakes with column subtraction. These are not 'careless mistakes' but fundamental misconceptions.
When subtracting with decimals such weaknesses are highlighted because of the decimal point. Children are uncertain about the order of operations when carrying out calculations.
Pupils are unable to accurately estimate and use the inverse to check.

Learning objectives (see overleaf for exemplification)

To solve any subtraction with numbers to 2 decimal places. (see y5 progression)
To carry out calculations involving the four operations.
To work systematically to solve a problem
To use estimation to check answers to calculations.
To subtract negative numbers.
To understand the order of operations using brackets.
To round numbers accurately
To solve multi step word problems.

Example Questions

Respond rapidly to oral explaining strategy: 750-255, take 300 from 1240, 3500 subtract 2050, subtract 2250 from 8500, 1700 less than 2500, 3000 less than 10220, what must I take from 8.4 to leave 2.6?
What is the difference between 2.2. and 6.5? How much more than 23.4 is 24.9? How much less than 6.8 is 4.2? What must I add to 5.4 to make 9.3? Decrease 5.6 by 1.9, 2.8 add a number is 4.3 what is the number? Find pairs of numbers with a difference of 13.5.
□-2.56=5.38, 7.65-6.85=□, □-1475=2924, □-Δ=0.03,
421.3- 82.57=□
Find the missing number in □-2485=4128
Vijay makes a sequence of numbers. He chooses a starting number and then subtracts equal amounts each time. The third number in his sequence is 45. The tenth number is -32. What is the first number in the sequence?
What number is 8 less than -4?
100-(22.75 + 19.08) = □

Mental strategies

(building on mental strategies from y5)

To find the difference by counting up through the next multiple. (count up from the smaller to larger number mentally: 8000-2785 is 5 +10+ 200+ 5000 = 5215
Subtract 0.9, 1.9, 2.9 or 1.1, 2.1, 3.1 by subtracting 1,2,3 then adjusting by 0.1
Work out mentally one fact 4.97-1.58 and then state three other related facts
Subtract four digit+ multiples of 100 (570,000 + 250,000= □)
Find what to add to a decimal with units, 10ths and 100ths to make the next higher whole number or 10^{th}. Subtract a pair of decimal fractions each less than 1 and with up to two decimal places.
Subtract numbers with different numbers of digits. Find the difference between 4387 and 782,175

Year 6 – Subtraction Progression (When planning ensure you track back to year 5)

To solve any subtraction with numbers to 2 decimal places.	**To carry out calculations involving the four operations.**	**To work systematically to solve a problem**

3.24 – 1.06= 2.18

3 ones 2 tenths 4 hundredths
3 ones 1 tenth 14 hundredths

First subtract the hundredths

3 . ¹2 ¹4
- 1. 0 6
8

14 hundredths – 6 hundredths = 8 hundredths
Then subtract the tenths:

3 . ¹2 ¹4
- 1. 0 6
. 1 8 (1 tenth – 0 tenths = 1 tenth)

Lastly subtract the ones:

3 . ¹2 ¹4
- 1 . 0 6
2 . 1 8

3 ones – 1 one = 2 ones

9 - 4 + 3 =

Calculating from left to right:

9 - 4 + 3 = 5 + 3 = 8

To use estimation and rounding to check answers to calculations.

By rounding the actual values to more manageable numbers, you can estimate the answers to many problems:

£2.99 + £3.10 + 99p ≈ £3 + £3 + £1 = £7

29 × 9 ≈ 30 × 10 = 300

61 ÷ 6 ≈ 60 ÷ 6 = 10

Arrange the numbers below in the circles so that the sum of the three numbers along each line is 4.5

1.2 1.8 1.4 1.6 1.5

To subtract negative numbers.

(-3) - (-2) = -1

-2

-10 -5 0

To understand the order of operations using brackets.

There were 94 players in Arsenal juniors. Last year 21 players left and 39 joined.
How many players are there now in Arsenal Juniors?

94 – 21 + 39
73 + 39

= **112** players are now in Arsenal juniors.

How would you use a bar model to represent this word problem?

To solve multi step word problems

Using all four numbers exactly once and any of the operations (including one pair of brackets). Make a number sentence that has a value of 1.

Here is one solution: 6 + 5 - 9 - 1 = 11

Can you make number sentences that have values from 2 to 20?

Chapter 4

Multiplication

EYFS 2 – Multiplication (When planning ensure you track forwards to year 1)	
Early Learning Goal 11 **Using quantities and objects, they add two single-digit numbers and count on or back to find the answer. They solve problems, including sharing, doubling and halving.**	KS1 ready: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of. **Key concepts**

Multiplication begins with counting patterns and contexts involving equal groups.
Objects can be added over and over again to make 'more'.
Objects can be sorted into groups of the same number.
To get the total you count according to the number in the group.
All steps need to be taught through play as they are the precursors to multiplicative reasoning. Children will be working in the concrete before moving towards the pictorial and abstract.

Learning objectives (see over page for exemplification)

To set out groups and find the total amount
To count patterns
To double numbers to 5, then 10
To double quantities of objects
To skip count in 2s
To skip count in 5s
To skip count in 10s
Begin to understand odd and even
Begin to relate the addition of doubles to counting on. (How many wheels on two cars? **4** (hold four in head) 5, 6, 7, **8** (count on). 4+4=8

Potential barriers/misconceptions

Children inaccurate when displaying arrays of cubes/objects and so pattern is not clear.
Link not clear between the array and the seemingly abstract number given as the answer.
Children unable to place objects in equal groups.
Not secure with one to one correspondence counting in ones, therefore will be unable to count pairs accurately
When counting orally in 10s: 60,70,80 follow a regular pattern which link to single digit numbers however 10, 20, 30 do not.
Conceptual understanding of 'same' and 'different' is not secure (both language and concept).

Example Questions

Give everyone two biscuits from the jar.
Can you count the spots on each side of your butterfly? Does it have an equal number of spots on each side? I will clap where there is a number missing. 1, 2, 3, (clap) 5. Tell me the missing number
2, 4, 6, (clap), 10 Tell me the missing number
10, 20, 30, (clap), 40. Tell me the missing number
How many fingers are there on two hands?
How many eggs are there in the box? How are they arranged? (in 2s)
Count the pairs of animals on the ark.
Count these pairs of socks. How many pairs are there? How many socks are there altogether?
How many buttons are there on this coat? Count them in twos. Now count them in fives. (answer-10)

Mental Maths (can revisited throughout day once concept has explicitly shared)

Count in tens (recite the sequence ten, twenty, thirty… one hundred.) Do the same backwards. Count on and back in tens from a given tens number
Say the tens number that goes before or after a given tens number. (When you count in tens, what number comes before 60? 90?)
Count from a given tens number and stop at another. (count on in tens from 20 and stop at 70, count back in tens from 60 and stop at 30)
Count around in a circle of children, starting with Abdul on 20, who do you think will say 70? Understand odd and even numbers linked to getting 'into pairs'.
Count pairs: children, socks, animals in the ark, eggs in an egg box

EYFS 2 – Multiplication Progression (a combination of these models and images can be used for every objective)

To set out groups and find the total amount	To find matching groups that are the same	To recognise when each person is given the same amount	To double numbers to 5, then 10

Can recognise when objects are put into groups of the same amount.

Double 1 is two

Double 4 is eight

To double quantities of objects

Using these toy chicks:

Can you double the chicks
In the first box?

How many chicks now?

Can you double the cubes?

To skip count in 2s

0 2 4 6 8

Count: Pairs of socks, pairs of shoes, pairs of gloves, pairs of animals, talk partners etc.

To skip count in 5s

0 5 10 15 20 25

To skip count in 10s

0 10 20 30 40

To place objects into arrays

Begin to relate the addition of doubles to counting on

How many wheels on two cars?

4 5, 6, 7, **and 8**.

4+4=8 Double 4 = 8

Begin to understand odd and even

Objects that can be counted in pairs are even

Objects that can't be counted in pairs areodd

Year 1 – Multiplication

(When planning ensure you track back to Reception and forwards to year 2)

National Curriculum

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Notes and guidance (non-statutory)

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.

Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, x, times, multiplied by, multiple of, one, twice, three times, ten times, repeated addition, array, row, column. **Key concepts**

Multiplication begins with counting patterns and equal groups in context.
Multiplication is introduced when objects or numbers are combined.
It is associated with the idea of repeated addition.
Pupils will be able to conceptualise multiplication as groups of items.

Potential barriers/misconceptions

Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary for example 'multiplied by'.
When objects placed in arrays it may be done inaccurately therefor link between arrays and answers unclear. Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones.
Don't understand how 'turning the grid around' shows that multiplication can be done in any order.

Example Questions

How many pencils do I need if everyone has to have 2? (There are eight people in the group) How many shoes do we need for these three dolls?
John makes biscuits with 1 egg, 4 spoons of flour, 3 spoons of sugar, 5 spoons of milk. Lucy makes double the amount of biscuits. She will need: □ egg, □ spoons of flour, □ spoons of sugar, □ spoons of milk. Add more fives until the total is 25. 5 + 5 + …….
Write the answer: 6 x 2 =

Learning objectives (see over page for exemplification)

To place objects into equal groups
To double numbers
To double two digit numbers
To place objects into arrays
To pictorially represent multiplication sentences
To understand repeated addition
Can describe an array in two ways
To make multiplication stories
To move towards the bar model to solve word problems

Mental Maths

To count in twos, fives and tens
Count forwards and backwards in 2s from any given number.
Count forwards and backwards in 5s from any given number.
Count forwards and backwards in 10s from any given number.
Recognition of all odd and even numbers
Rapid recall of doubles to 10 (and corresponding halves)
Rapid recall of doubles to 20

Year 1 – Multiplication Progression (a combination of these models and images can be used for every objective)

To place objects into equal groups

How many pencils in each group? Adding the same number: How many groups of shells arethere? How many shells are there in each group? There are 4 groups
Each group has 3 shellsin.
3 + 3 + 3 + 3 = 12
(4 threes = 6)
How many altogether? (3 groups of 4 = 12)
Can you count in 5s? **There are 12 shells altogether.**

To double numbers

To double numbers (over 10)

12

10 + 2

Double 10 Double **2**
20 + 4 = 24

To place objects into arrays

Arrange your objects into rows
you arrange them in different ways?
Can
Each row must have the same

To pictorially represent multiplication sentences

5 x 2 = 10

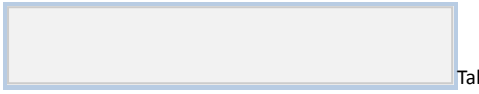
5 + 5 = 10

number.
Can you add the total amount?

To understand repeated addition

To describe an array in two ways

To make multiplication stories



Take note of the number of groups first Can you tell a multiplication story aboutthese cakes? Then the number of items in each group.

I see □ groups of cakes.
Each group has □ cakes. □ x □ = □ There are □ cakes altogether.

To move towards the bar model to solve word problems

There are three children. Each child has five sweets. How many sweets do they have altogether?

?

5 + 5 + 5 = 15

5 x 3 = 15

3 x 5 = 15

They have 15 sweets altogether

Year 2 – Multiplication (When planning ensure you track back to year 1 and forwards to year 3)

National Curriculum

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other.

They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).

Key vocab: count in, double, halve, lots of, groups of, times, group in pairs, equal groups of, x, times, multiplied by, multiple of, one, twice, three times, ten times, times as (big, long, wide… and so on) repeated addition, array, row, column.

Key concepts

Key vocabulary: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide…) X

Multiplication is introduced when objects or numbers are combined.

It is associated with the idea of repeated addition.

Pupils will be able to conceptualise multiplication as groups of items.

Multiplication can be done in any order- this can be shown in the arrangement of arrays. Doubling is the inverse of halving

Potential barriers/misconceptions

Pupils may not focus on 'rows of' or 'columns of' but only see arrays as a collection of ones.

Don't understand how 'turning the grid around' shows that multiplication can be done in any order. Not understanding that multiplication is repeated addition

Learning objectives (see over page for exemplification)

To identify odd and even numbers

To understand multiplication as repeated addition

To use arrays

To know 2, 5, 10 timestables.

To multiply usingpartitioning

To understand the commutative property of multiplication.

To interpret multiplication sentences (The first factor referring to the number of groups and the second factor as the number of items in each group.)

To know all corresponding multiplication and division facts (i.e. 2x4= 8, 4x2= 8 and 8÷4 =2, 8÷2=4) To break a number into factors

To connect the 10 times table with place value

To use the bar model to represent word problems

Example Questions

Respond rapidly to oral and written questions such as: two fives, double 5, 6 times 2, 5 multiplied by 2, multiply 4 by 2. Two tens, double 2, 3 times 4, 9 multiplied by two, multiply 5 by 8. Is 20 a multiple of 5?

6x2=□ 9x□=18 □ x2=14

6x10=□ 2x□=20 □x10=40

How many wheels are there on three cars?

Jo's plane is 6cm wide. Mo's box is twice as wide. How wide is Mo's box? (scaling) Ella's dad washes some cars. He uses 12 buckets of water. Each bucket has five litres of water. How many litres of water does he use altogether?

Tara does not know how to work out 16 x 5. Can you show her how to do this?

There are 15 apples in a tray. Ling has 4 trays of apples. How many apples does Ling have altogether? Show how you work it out.

Mental Maths

Rapid recall of 2,5 and 10 times tables

Count in 5s clockwise around a clock face/ anticlockwise around a clock face.

Count forwards and backwards in 2s, 5s and 10s from any given number.

Recognition of all odd and even numbers

To recall related multiplication and division facts linked to other multiplication tables. (3x4=12, 4X3=12, 12÷4=3, 12÷3=4) Rapid recall of doubles and their corresponding halves. (double 12 is 24, half 24 is 12)

Rapid recall of half of all 2 digit even numbers. (half of 12, 18, 42 etc)

Recognise that multiples of 10 end in 0, 5 end in 5 and 0, 2 end in 0,2,4,6,8.

Recognise two digit multiples of 10,5,2 (65 is a multiple of 5, 72 is a multiple of 2, 50 is a multiple of 5 and 10) Work out the four times table by doubling the two times table.

Multiply a single digit by 1 or 10. (3x1 = 3, 7x10=70 etc)

Multiply a single digit up to 5 by 2,3,4,5. (2x3=□ 4x4=□)